Grade 3 Operations and Algebraic Thinking Unit of Instruction

This is a progressive unit of instruction using the Concrete-Representational-Abstract (CRA) Instructional Model. CRA is a three-part instructional model that begins by using concrete materials, then progresses to representational pictures and finally abstract notation. This unit is not intended to replace your district's curriculum, but rather it serves to support the teaching and learning of the third grade operations and algebraic thinking standards. In this unit, students will begin by investigating the standards while using manipulatives to explore the concepts. Then, students will represent their learning through pictures, visuals and drawings. Finally, students will demonstrate their understanding through abstract notation and algorithms. This unit of study will cover the following standards MAFS.3.OA.1.1, MAFS.3.OA.1.2, MAFS.3.OA.1.3, MAFS.3.OA.1.4, MAFS.3.OA.2.5, MAFS.3.OA.2.6, MAFS.3.OA.3.7, MAFS.3.OA.4.8 and MAFS.3.OA.4.9.

The unit begins with a list of resources and tools to assist in teaching operations and algebraic thinking. Then, each of the grade three operations and algebraic thinking standards is listed along with aligned instructional resources and formative assessments. The component of CRA is identified for each of the resources and formative assessments. The resources presented in this document may only cover portions of the aligned standard and represent a small sample of those available on CPALMS.

The Mathematical Practices are habits of mind that describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Practices should be infused during the course and will be assessed throughout the Grade 3 Mathematics FSA. More information about each Mathematical Practice can be found by clicking on the links below.

MAFS.K12.MP.1.1 Make sense of problems and persevere in solving them.
MAFS.K12.MP.2.1 Reason abstractly and quantitatively.
MAFS.K12.MP.3.1 Construct viable arguments and critique the reasoning of others.
MAFS.K12.MP.4.1 Model with mathematics.
MAFS.K12.MP.5.1 Use appropriate tools strategically.
MAFS.K12.MP.6.1 Attend to precision.
MAFS.K12.MP.7.1 Look for and make use of structure.
MAFS.K12.MP.8.1 Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking

A bibliography of children's literature with a focus on operations and algebraic thinking is provided. These books can be integrated into the lessons to connect mathematics and literature.	1. *Amanda Bean's Amazing Dream*, Cindy Neuschwander 2. *The Doorbell Rang*, Pat Hutchins 3. *Each Orange Had 8 Slices,* Paul Giganti 4. *The Grapes of Math,* Greg Tang 5. *The Hershey's Multiplication Book,* Jerry Pallotta 6. *The Lion's Share,* Matthew McElliot 7. *Sea Squares,* Joy Hulme 8. *One Hungry Cat*, Joanne Rocklin & Rowane Murphy 9. *2 x 2 = Boo!,* Loreen Leedy
3rd Grade Mathematics Course Description	Course descriptions provide an overview for a course and designate which standards are in that course. The course description includes resources for all 39 standards within the 3rd grade mathematics course.
Hooray Arrays Déjà vu Addition Array Addition I Array + You Array = Arrays! *Lesson Plans* Concrete-Representational-Abstract	This sequence of four lessons is designed to guide students through an introduction to multiplication. Students learn to recognize repeated addition as multiplication through the use of visual models. Students learn how arrays can be used to represent equations.
Test Item Specifications	The Test Item Specifications indicate the alignment of items with the Florida Standards. Assessment limits are included in the specifications, which define the range of content knowledge in the assessment items for the standard. Sample items for each standard are also included in the specifications document.
Test Design Summary and Blueprint	The Test Design Summary and Blueprint shows the reporting categories with a corresponding weight for the 3rd Grade Mathematics FSA.
Florida Students	Resources specifically designed with students in mind are available on Florida Students. Florida Students is an interactive site that provides educational resources aligned to the Florida Standards.
3rd Grade Mathematics Parent Guide	The parent guide will support parents and families with children in Grade 3 Mathematics.

Instructional Resources

MAFS.3.OA.1.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.

Cheezy Arrays *Lesson Plan* Concrete	This lesson is a hands on activity that includes multiplication using arrays. The lesson also serves as a great transition from repeated addition to multiplication.
Hip, Hip, Array! *Lesson Plan* Concrete-Representational	Students will develop their understanding of arrays by building arrays using hands on activities. Students will have the opportunity to play a game to practice their skills with arrays.
Just Group It *Lesson Plan* Concrete-Representational	Students will be able to use counters to create equal groups and write multiplication sentences.
Circles and Stars *Lesson Plan* Representational	This is an introductory lesson to prepare students to move from using repeated addition to using grouping of numbers when solving multiplication problems. After completing this lesson, students would then begin a lesson using arrays for equal groups.
How Many Circles? How Many Stars? *Lesson Plan* Representational-Abstract	This lesson will give students a visual representation of multiplication and repeated addition. It will also help students see multiplication as the combining of equal-size groups that can be represented with a multiplication equation.
Let's Multiply Using Groups and Arrays *Lesson Plan* Representational-Abstract	In this lesson, students will use pictures in arrays and groups to write multiplication story problems, write equations, solve the problems and identify the factors and products.
"Shoot for the Stars" to Introduce Multiplication *Lesson Plan* Representational-Abstract	Students will learn how multiplication is related to repeated addition and how to use different strategies to find the product.

Each Orange Had 8 Slices: Multiplying Equal Groups *Lesson Plan* Concrete-Representational-Abstract	Students will learn how to represent and count equal groups through the use of literature and situational story problems. Using the story *Each Orange Had 8 Slices*, students will use manipulatives to create arrays to calculate.
Arrays Show the Way to the Multiplication Chart *Lesson Plan* Concrete-Representational-Abstract	Students build arrays and save the arrays in a class Multiplication Chart. They learn to use arrays to find products and factors. By placing the arrays in the Multiplication Chart, they learn how to read the chart. They learn how to write equations relative to the number of rows and items in the row.

Formative Assessments

Multiplication on the Number Line Representational-Abstract	Students are asked to explain how to use a number line for multiplying, in the context of a word problem.
Interpreting Multiplication Abstract	Students are asked to explain what 5 x 7 means and to provide a real-world context for 5 x 7.
What Does the 21 Mean? Abstract	Students are given a context for a multiplication problem and asked to determine how to solve it and what the product means.
Writing Multiplication Word Problems Abstract	Students are asked to write multiplication word problems prompted by pictures and then to write both an addition and a multiplication expression that can be used to solve the problem.

Instructional Resources

MAFS.3.OA.1.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

Pet Store Partitive Division *Lesson Plan* Concrete-Representational-Abstract	In this lesson students will model partitive division through the real-world activity of a pet store owner.
Everybody Wins! *Lesson Plan* Concrete-Representational	The lesson in narrative form gives students experience with the partitioning (sharing) model of division. The context for the lesson is Sheila Bruce's book *Everybody Wins!,* in which Oscar, Emmy, Hugo, and Tony calculate how to share pizzas, baseball tickets, bubble gum and more.
Solve Division Problems by Subtracting Equal Groups *Tutorial* Representational-Abstract	In this lesson, the learners will learn how to solve division problems by repeatedly subtracting equal groups.

Formative Assessments

Using a Number Line to Solve a Division Problem Representational-Abstract	Students are asked to explain how to use a number line for dividing, in the context of a word problem.
What Does the 6 Mean? Abstract	Students are given a division word problem and asked to determine how it might have been solved and what the quotient means.
Writing a Problem with a Quotient Abstract	Students are asked to solve a division equation and then interpret the quotient by writing a word problem that can be modeled by the equation.

Instructional Resources

MAFS.3.OA.1.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Bridging Addition and Multiplication with Word Problems *Lesson Plan* Representational	This introductory lesson uses word problems to help students make connections between arrays and multiplication with single and/or two-digit quantities within 100.
Array to Multiply *Lesson Plan* Representational-Abstract	In this lesson, students will understand the concept of arrays and will be able to create and draw an array for multiplication sentences. An engaging center game is used to help students with the concrete and visual representation of multiplication sentences.
Apples, Oranges, and Bananas of Math? *Lesson Plan* Representational-Abstract	In this lesson, the students will work independently or in small groups to write their own math riddles around the concepts of multiplication. The teacher will use the book, *The Grapes of Math* by Greg Tang, to support this lesson.
Chip Chip Array! *Lesson Plan* Representational-Abstract	This lesson will have students working together to create arrays to represent given multiplication problems.
Array Frame, Your Best Friend *Lesson Plan* Representational-Abstract	In this lesson, students will learn to use the structure of array frames to build familiarity and fluency with the array as a tool. Students will solve several multiplication word problems using the array as a representation.
Make Your Way with Arrays *Lesson Plan* Concrete-Representational-Abstract	Students will solve multiplication and division word problems by drawing arrays and writing the related equation.
Magnificent, Amazing Multiplication Arrays! *Lesson Plan* Concrete-Representational-Abstract	In this lesson students will be creating, describing, and analyzing arrays as they relate to multiplication. In addition, they will be working on hands-on activities that allow them to justify their answers and prove their reasoning.

Formative Assessments

Finding an Unknown Product Abstract	Students are asked to model equal groups and an array problem in which the product is unknown with multiplication or division equations and then solve each problem.
Finding the Group Size Abstract	Students are asked to model equal groups and an array problem in which the group size is unknown with multiplication or division equations and then solve each problem.
Finding the Number of Groups Abstract	Students are asked to model equal groups and an array problem in which the number of groups is unknown with multiplication or division equations and then solve each problem.
Measurement Problems Abstract	Students are asked to model a multiplication and a division problem that involve measurement quantities with multiplication and division equations and then solve each problem.

Instructional Resources

MAFS.3.OA.1.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = [] ÷ 3, 6 × 6 = ?.*

Discovering the Mystery Factor Through Arrays *Lesson Plan* Concrete-Representational	Students will begin with the use of manipulatives to solve for unknown factors by building arrays. They will progress to drawn models as mastery is shown with manipulatives.
Giddy Up, Round Up: Relating Division to Multiplication *Lesson Plan* Representational-Abstract	In this lesson, students will learn to solve division problems by relating them to multiplication facts. Practice materials focus on the 6's and 8's multiplication facts.
Tasty Algebra: Using Cereal to Find Missing Factors *Lesson Plan* Concrete-Representational-Abstract	In this lesson students will use cereal pieces to solve multiplication equations that include missing factors ranging from one through ten. Students will also argue the validity of multiplication equations that include missing factors and products with corresponding word problems.

Formative Assessments

Find the Unknown Number Abstract	Students are given multiplication and division equations within 50 and are asked to find missing numbers. The missing numbers are presented in all positions.
Missing Numbers in Division Equations Abstract	Students are given division equations involving numbers within 50 and are asked to find missing numbers. The missing numbers are presented in all positions.
Multiplication and Division Equations Abstract	Students are given multiplication and division equations within 100 and are asked to find the missing numbers. The missing numbers are presented in all positions.

Instructional Resources

MAFS.3.OA.2.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

Partial Products with Arrays *Lesson Plan* Concrete-Representational	In this inquiry based lesson, students will find partial products of arrays to solve multiplication problems. They will also move from a concrete representation of arrays using manipulatives to a pictorial representation.
Candy Apple Fun *Lesson Plan* Representational-Abstract	In this lesson, students will learn how to break down simple multiplication problems by using arrays to model the Distributive Property.
Decomposing Arrays *Lesson Plan* Representational-Abstract	This lesson introduces students to the concept behind the Distributive Property of Multiplication. Students will decompose arrays into smaller parts and write the corresponding multiplication sentence for each smaller array.
One with a Bun *Lesson Plan* Representational-Abstract	In this lesson students will explore the Multiplicative Identity Property of 1, using array and equal-group models. Students will model story problems, translate problems into equations and identify patterns to develop understanding of the Multiplicative Identity Property of 1.
Zero on a Hero *Lesson Plan* Representational-Abstract	Students will explore the Zero Property of Multiplication using array and equal-group models for multiplication. Students will model story problems, translate problems into multiplication facts and identify patterns to develop understanding of the Zero Property of Multiplication.
Amazing Arrays *Lesson Plan* Concrete-Representational-Abstract	This is a hands-on lesson for introducing and practicing building arrays to create models that represent the distributive property of multiplication, and then using those arrays to draw models of the equations they represent.

Fall Fun and Games Using the Commutative Property *Lesson Plan* Concrete-Representational-Abstract	In this lesson, students will build and manipulate a variety of arrays in the context of creating games for a Fall Festival. They will practice using the Commutative Property of Multiplication to find related multiplication facts.
Checking ID's *Lesson Plan* Concrete-Representational-Abstract	Students will learn, understand and use the Identity Property of Multiplication as well as the Zero and Commutative Properties to solve multiplication problems involving factors of 0 and 1.
Hungry Zero *Lesson Plan* Concrete-Representational-Abstract	The definition of the Zero Property of Multiplication will be presented and discussed, along with a review of other vocabulary necessary for this lesson (factors, products, groups of, Commutative Property). The lesson will show the process of multiplying by 0.

Formative Assessments

Break Apart and Put Together Representational-Abstract	Students are given two arrays, one representing the equation 7 x 9 = 63 and the other representing the equation (5 + 2) x 9 = 63, to see if they recognize a relationship between the two.
Does It Work for Division? Abstract	Students are asked if the Commutative Property holds for division.
Meeting the Reading Goal Abstract	Students are given two problems to solve, one represented by the equation 4 x 6 = 24 and the other by the equation 6 x 4 = 24, to see if they recognize the answer to the second problem based on the Commutative Property.
Using the Associative Property of Multiplication Abstract	Students are asked to find the product of three numbers and are observed to see if they use the Associative Property to find the product more easily.

Instructional Resources

MAFS.3.OA.2.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Grandma Wants to Know! *Lesson Plan* Concrete-Representational-Abstract	Help Mom and Dad tell Grandma about Cindy's trip to the carnival using bar models and arrays to relate division to multiplication with an unknown factor.
Three is NOT a Crowd! *Lesson Plan* Concrete-Representational-Abstract	This lesson will provide students with practical application activities to help them understand how division is simply solving a problem with an unknown factor. Students will be given snacks in which they must share with their group members equally.
Unknowns with Multiplication and Division Equations *Lesson Plan* Abstract	In this Khan Academy tutorial video, students will find the number to replace the symbol for the unknown in multiplication and division equations.

Formative Assessments

Alien Math Abstract	Students are told of a planet where division is not taught, and asked to rewrite division problems as multiplication problems to solve. The students are also asked to explain why it might be easier to solve as multiplication.
Changing Division Equations into Multiplication Equations Abstract	Students consider a division fact and are asked to turn it into a multiplication fact. If successful, they are asked to rewrite a basic division fact that they are not likely to know and which has a symbol for the unknown number.
Multiplication as the Inverse of Division Abstract	Students are given a word problem and asked to write an equation for the problem. Then the students are to select a multiplication equation that can also be used to solve the problem.
Using Multiplication to Solve Division Problems Abstract	Students are asked to solve a division problem using a calculator but without using the division key.

Instructional Resources

MAFS.3.OA.3.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Introduction to Multiplication Using Literature *Lesson Plan* Concrete-Representational	This lesson plan can be used to introduce the concept of multiplication to students through the use of literature. The story *Amanda Bean's Amazing Dream* is used to demonstrate the different ways to count items and how multiplication can make that process much faster.
Arithmetic *Virtual Manipulative* Representational-Abstract	The students will be given multiplication and division problems which they must answer. They also have the option of being given a number then stating the factors of how that number was attained using either multiplication or division.
Skip Counting to Multiply *Lesson Plan* Abstract	Students will build a conceptual understanding of multiplication by creating a hundreds chart, using different colors to assist them with skip counting by 2, 3, 5 and 10. Students will discuss, "How many groups of (2, 3, 5 and/or 10)?" are in each number.
Product Game *Virtual Manipulative* Abstract	This interactive game for two players develops students' fluency with multiplication facts, their understanding of the relationship between factors and products and their strategic thinking.
Four in a Row Multiplication Game *Educational Game* Abstract	In this interactive game, players use multiplication facts to cover four squares in a row. Players must change one number from the previous player's turn to create a new fact. The products from each turn of play are covered on the game board.
Alien Munchtime Division Game *Educational Game* Abstract	This is an online game where students review division facts while feeding hungry aliens. The user can set the amount of fact families they want to play with from 2 to 12.
Arithmetic Workout *Educational Game* Abstract	This interactive Flash applet offers three ways for students to practice basic fact skills related to multiplication and division.

Making Sense of Multiplication to Build Fluency *Lesson Plan* Concrete-Representational-Abstract	This lesson will help students multiply numbers with factors of 6, 7, 8 or 9 through decomposing numbers in an array and applying the distributive property. Teaching students how to use an array can give them a visual representation of the final product.

Formative Assessments

Fluency with Basic Multiplication Facts Abstract	Students are assessed on their fluency with multiplication facts for the products of two one-digit numbers.
Fluency with Division Abstract	Students are assessed for fluency with division.
Fluency with Multiplication Abstract	Students are assessed for fluency with multiplication.
Using Flexible Strategies Abstract	Students are asked to describe two different ways to find the product of two numbers.

Instructional Resources

MAFS.3.OA.4.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Getting the Hang of Two-Step Word Problems *Lesson Plan* Representational-Abstract	Students will solve two-step word problems involving subtraction and division and represent these problems using equations with a letter standing for the unknown quantity.
Multiplying Numbers: Fall Festival Extravaganza *Lesson Plan* Representational-Abstract	Students will organize and analyze data to create equations with variables. Students will use their understanding of equations to solve two-step word problems using data collected from the Morgan Woods Fall Festival Supply Sheet.
The Stamp Collection *Problem-Solving Task* Abstract	The task will have students create an equation to represent a multistep word problem and solve the problem using a variety of operations.

Formative Assessments

Bake Sale Abstract	Students solve a two-step word problem involving addition and division and then write an equation to represent the problem.
Books at the Book Fair Abstract	Students solve a two-step word problem involving multiplication and subtraction and then write an equation to represent the problem.
Zoo Field Trip Abstract	Students solve a two-step word problem involving subtraction and division and then choose an equation that represents the word problem.

Instructional Resources

MAFS.3.OA.4.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

The Power of Patterns *Lesson Plan* Representational-Abstract	Students will solve a real world problem to discover similarities and differences between the patterns of adding 2 to a number and doubling a number. The problem is set in the real world context of twin brothers who choose different patterning strategies given by their grandma to save for buying a car.
Tricky Rice Math Patterns *Lesson Plan* Representational-Abstract	This is a 3rd grade MEA that requires students to use mathematical patterns to solve the problem, along with the analysis of data. After reading *One Grain of Rice* by Demi, students will look for ways to help Rani's relative find a new pattern.
Number Line Bars *Virtual Manipulative* Representational-Abstract	A versatile tool that can be used to illustrate the operations of addition, subtraction, multiplication and division.
Addition Patterns *Problem-Solving Task* Representational-Abstract	The purpose of this task is to study some patterns in a small addition table. Each pattern identified persists for a larger table and if more time is available for this activity students should be encouraged to explore these patterns in larger tables.
Symmetry of the Addition Table *Problem-Solving Task* Representational-Abstract	The goal of this task is to help students understand the commutative property of addition by examining the addition facts for single digit numbers. This is important as it gives students a chance to do more than memorize these arithmetic facts, which they will use throughout their education.
Making a Ten *Problem-Solving Task* Representational-Abstract	This task asks students to study more carefully the make-a-ten strategy that they should already know and use. In this strategy, knowledge of which sums make a ten, properties of addition and subtraction and identifying patterns are used to evaluate sums which are larger than 10.
Patterns in the Multiplication Table *Problem-Solving Task* Representational-Abstract	The goal is to look for structure and identify patterns and then try to find the mathematical explanation for this. This problem examines the checkerboard pattern of even and odd numbers in a single digit multiplication table.

Formative Assessments

Adding Odd Numbers Abstract	Students are asked to consider what type of number results when adding two odd numbers and when adding three odd numbers.
Adding Odds and Evens Abstract	Students are asked to consider the parity of the sums of two even numbers, two odd numbers and an even and an odd.
Decomposing Into Equal Addends Abstract	Students are presented with an equation and asked to find a pattern within the equation and to determine if the equation is true or not.
Multiplication of Even Numbers Abstract	Students are asked to determine if the total number of students in five classes will be even or odd.
Patterns with the Multiplication Table Abstract	Students are asked to find the missing numbers in a column of a multiplication table by using a pattern found within the table.