

# **Mathematics**

## **6–12**

### **Section 26**

# Mathematics 6–12

## 1 Knowledge of algebra

1. Apply the properties of real numbers: closure, commutative, associative, distributive, transitive, identities, and inverses.
2. Solve linear equations and inequalities in one or two variables, symbolically or graphically.
3. Relate the graphical and algebraic representations of linear equations or inequalities on a number line or in the coordinate plane.
4. Determine the slope, intercepts, or equation of a line, given appropriate information.
5. Formulate and solve systems of linear equations or inequalities, including models of real-world situations.
6. Solve equations or inequalities involving absolute value.
7. Solve or identify models of real-world problems involving ratio, proportion, or direct, inverse, and joint variations.
8. Solve or identify equations containing rational or radical expressions including models of real-world situations.
9. Solve quadratic equations using factoring, graphing, completing the square, or applying the quadratic formula, including complex solutions.
10. Solve or identify models of real-world problems using quadratic equations.

## 2 Knowledge of advanced algebra

1. Relate graphical and algebraic representations of nonlinear inequalities.
2. Perform arithmetic operations of complex numbers algebraically or geometrically.
3. Solve equations involving radicals (index greater than two), powers, exponents, and logarithms.
4. Solve polynomial equations (third degree or higher).
5. Expand binomials to a positive integral power or determine a specified term in the expansion.
6. Perform and model vector addition, subtraction, and scalar multiplication on the plane.
7. Determine a specified term or a rule for an arithmetic or geometric sequence.

- Determine the sum of terms in an arithmetic or geometric sequence.
- Perform operations on matrices.
- Solve and interpret models of real-world problems using discrete structures (e.g., sequences, finite graphs, matrices).

### **3 Knowledge of functions**

- Determine which relations are functions, given mappings, sets of ordered pairs, rules, and graphs.
- Determine the domain and range of a given function.
- Relate graphs and equations of functions (e.g., absolute value, step, piecewise, polynomial, exponential, periodic).
- Determine the inverse of a given function or the composition of two functions and their related properties.
- Determine or prove whether a function is symmetric, periodic, or even/odd.
- Determine the graph or equation of a transformed function.

### **4 Knowledge of geometry**

- Identify or apply the relation between the perimeter, area, or volume of similar figures.
- Determine the relationships between points, lines, and planes, including their intersections with other two and three dimensional figures.
- Differentiate and relate geometric figures or solids according to their properties.
- Derive or apply formulas to find the measures of interior and exterior angles of convex polygons including their sum.
- Prove or apply properties of quadrilaterals, including models of real-world situations.
- Prove theorems or solve problems with similar or congruent polygons or solids.
- Apply the Pythagorean theorem or its converse, including models of real-world situations.
- Apply 30-60-90 or 45-45-90 triangle relationships to solve problems.
- Derive or apply formulas for perimeter, circumference, or area of two-dimensional figures (e.g., circles, sectors, segments, arc lengths, polygons, composite figures).
- Apply theorems pertaining to the relationships of chords, secants, diameters, radii, and tangents with respect to circles and to each other.

11. Prove or apply theorems pertaining to the measures of inscribed angles and angles formed by chords, secants, and tangents.
12. Analyze basic geometric constructions (e.g., bisecting angles or line segments, constructing parallels or perpendiculars).
13. Derive or apply formulas for surface area and volume of prisms, pyramids, cylinders, cones, and spheres.

## **5 Knowledge of coordinate geometry**

1. Apply distance or midpoint formulas involving two points, a point and a line, or two parallel lines.
2. Determine the equation or graph of parabolas, ellipses, and hyperbolas, and the directrix, foci, vertices, axes, and asymptotes, given appropriate information.
3. Determine the equation, graph, center, or radius of a circle, given appropriate information.
4. Determine the images of geometric objects on a coordinate plane under translations, rotations, dilations, or reflections, and analyze appropriate properties of images and preimages.
5. Determine the equation of a conic section to model real-world situations.

## **6 Knowledge of trigonometry**

1. Determine equations of graphs of circular/trigonometric functions and their inverses.
2. Prove circular/trigonometric function identities or apply them to solve problems.
3. Analyze the graphs of trigonometric functions (e.g., amplitude, period, phase shift).
4. Solve real-world problems involving triangles using the law of sines or the law of cosines.
5. Apply trigonometric ratios to solve right triangle problems.
6. Determine an equation to model real-world periodic phenomenon.

## **7 Knowledge of statistics and probability**

1. Interpret data on a single categorical or quantitative variable (e.g., measures of central tendency, spread, location).
2. Interpret data on two categorical or quantitative variables (e.g., correlation, linear regression, two-way tables) or identify an appropriate representation.

3. Interpret information from bar graphs, histograms, circle graphs, stem-and-leaf plots, scatter plots, and box-and-whisker graphs or identify an appropriate representation for given data.
4. Identify the processes used to design and conduct statistical experiments including possible sources of bias.
5. Infer and justify conclusions from sample surveys, experimental data, and observational studies.
6. Solve problems using the Fundamental Counting Principle, permutations, and combinations.
7. Determine probabilities of independent, dependent (i.e., conditional probability), or compound events using a variety of procedures (e.g., counting procedures, tables, sample spaces, tree diagrams, permutations, combinations).
8. Use probability to evaluate outcomes of decisions, including the use of expected value.

## **8 Knowledge of calculus**

1. Determine limits using theorems concerning sums, products, and quotients of functions.
2. Determine the derivatives of algebraic, trigonometric, exponential, and logarithmic functions.
3. Determine the derivative of the sum, product, quotient, or the composition of functions.
4. Determine the slope or equation of a tangent line at a point on a curve.
5. Use the first derivative of a given function in various representations to determine increasing and decreasing intervals or extrema.
6. Use the second derivative of a given function in various representations to determine intervals of concavity or points of inflection.
7. Solve problems using velocity and acceleration.
8. Solve problems using instantaneous rates of change and related rates of change.
9. Determine antiderivatives for algebraic, trigonometric, exponential, and logarithmic functions.
10. Solve distance, area, and volume problems using integration.
11. Relate functions and their derivatives through the use of graphs or tables.

## **9 Knowledge of mathematical reasoning**

1. Evaluate arguments or conjectures using laws of logic or counterexamples.

2. Identify or compare the converse, inverse, and contrapositive of a conditional statement.
3. Analyze mathematical assertions within proofs (e.g., direct, indirect, mathematical induction, algebraic).
4. Classify examples of reasoning as inductive or deductive.

## **10 Knowledge of instruction and assessment**

1. Analyze students' misconceptions and choose instructional approaches to address the misconceptions with particular focus on number operations, algebra, advanced algebra, functions, and geometry.
2. Identify or evaluate diagnostic, formative, or summative assessment items that measure conceptual or procedural understanding.
3. Determine the appropriate sequence of lessons for a specific mathematical concept.
4. Evaluate student responses to identify misconceptions or gaps in knowledge and provide specific feedback to promote student achievement, including the use of a rubric.
5. Analyze a given instructional segment which may include the selection or use of available tools and resources (e.g., mathematical models, manipulatives, dynamic grapher, interactive geometry drawing tool, computer algebra system, appropriate calculators).
6. Develop and interpret appropriate models for mathematical concepts including real-world models, and equivalent representations (e.g., graphical, symbolic, verbal, numeric).
7. Identify or apply methods, strategies, and questioning techniques for teaching problem-solving skills and applications (e.g., constructing tables from given data, guess-and-check, working backwards, reasonableness, estimation).
8. Differentiate among various learning environments, including alternative methods of assessment (e.g., performance, portfolios, projects) to accommodate the needs and diversity of students.