FCAT

Florida Comprehensive Assessment Test®

lessons learned

FCAT Mathematics Lessons Learned:

2001–2005 Data Analyses and Instructional Implications

999-8570-65-4

Acknowledgments

Great appreciation is extended to the following people for their contributions to the development of this document. The individuals listed below worked together to share their expertise and insights about student performance, to compile the information, and to prepare it for publication. We are grateful for the contributions of everyone involved in the development and production of *FCAT Mathematics Lessons Learned: 2001–2005 Data Analyses and Instructional Implications*.

Margaret Bambrick
Volusia County

Sandra Cook
Washington County

Stewart Elizondo
Marion County

Joyce Hawkins
Broward County

Sharon LaPoint
Indian River County

Wayne Moring
Retired
Escambia County

Charlene Richardson-Brinson
Polk County

Zulaima Sarmiento
Miami-Dade County

Pamela Scott
Leon County

Christopher Stevenson
Miami-Dade County

Gladys Thompson
Retired
Duval County

Pat Wells
Clay County

Richard Westover
Retired
Sarasota County

Ramona Wright
Escambia County

Florida Department of Education
Assessment & School Performance
Test Development Center

Harcourt Assessment, Inc.

Table of Contents

Index of Graphs

Index of Tables

Lessons Learned INTRODUCTION

Introduction to *FCAT Mathematics Lessons Learned: 2001–2005 Data Analyses and Instructional Implications*

Purpose

The purpose of the *FCAT Mathematics Lessons Learned* report on the Florida Comprehensive Assessment Test® (FCAT) is to provide a summary and analysis of the trends in student achievement of the Sunshine State Standards (SSS) in mathematics (Grades 3–10) from 2001 through 2005. The goals of this report are to inform education stakeholders and to provide guidance for educators that can be used to effect positive change and enhance program effectiveness. This is the second *Lessons Learned* report for FCAT Mathematics; the first report, released in January 2002, covered FCAT Reading, Writing, and Mathematics data from 1998 through 2000 and is available on the Florida Department of Education (DOE) website, http://fcat.fldoe.org/fclesn02.asp. A companion volume to this one, *FCAT Reading Lessons Learned: 2001–2005 Data Analyses and Instructional Implications*, is being released at the same time. The next volumes in the set will be *FCAT Science Lessons Learned* and *FCAT Writing Lessons Learned*. Unless otherwise indicated in this publication, *Lessons Learned* refers to this mathematics volume.

The information in this report provides Floridians who are interested in education with a comprehensive view of student achievement in Florida. Postsecondary educators working in teacher education will also benefit from the insights of this report. Other persons for whom these insights may be meaningful include parents, students, legislators, media representatives, and business organizations; however, the report places the highest priority on supporting those charged with improving student performance in Florida: teachers, administrators, curriculum specialists, school advisory councils, and district leaders.

Rationale

The phrase *lessons learned* implies a historical look at student achievement with thoughtful consideration given to how well students have learned the content of the assessed standards and how these results could be improved. This was accomplished by producing FCAT results and identifying trends, then convening a representative group of Florida educators to interpret the trends and identify instructional implications. The insights provided in this document may be used to identify and implement modifications in curriculum, instruction, and assessment practices in classrooms and schools throughout the state.

Historical Background

In 1996, the Florida educational community identified a core body of knowledge and skills that all Florida students should acquire. This body of knowledge, called the Sunshine State Standards, spanned seven content areas (language arts, mathematics, science, social studies, health and physical education, foreign language, and the arts). By adopting the Sunshine State Standards in May 1996, the Florida Board of Education defined a clear set of standards upon which to build an equitable system of student assessment and school accountability.

In 1995 and 1996, the Florida Educational Reform and Accountability Commission recommended the development of a statewide assessment system. These recommendations, called the Florida Comprehensive Assessment Design, led to development of the Florida Comprehensive Assessment Test® (FCAT). The development of FCAT Reading and FCAT Mathematics questions began in 1996, and questions were field tested in 1997. In 1998, the first results of the assessment were reported for students and schools. In 1999, the law related to student assessment was revised to require an annual assessment of all students in Grades 3–10. This legislation, called the A+ Plan, required that tests in reading and mathematics be developed for the grades not previously tested. Newly assessed grades for reading included Grades 3, 5, 6, 7, and 9; in mathematics, Grades 3, 4, 6, 7, and 9 were included in the state assessment.

In the 2001 FCAT administration, first steps were taken to create reading and mathematics developmental scale scores (DSS) that could provide educators with valuable information about student growth across grades. The DSS was successfully added as another measure of student achievement (in addition to scale scores) and is provided as one of the measures of student achievement in this *Lessons Learned* report. The DSS places scores for all grades on one vertical scale, allowing educators to track longitudinal growth and more accurately compare results across grades.

A detailed chronology of the FCAT program is included in the *FCAT Handbook—A Resource for Educators* published in 2005. The online version is available on the Department of Education's website at http://fcat.fldoe.org/handbk/fcathandbook.asp.

Criterion-Referenced Tests and Norm-Referenced Tests

The FCAT consists of two types of tests used to measure achievement and guide instruction of individual students. Criterion-referenced tests (CRT) in reading, mathematics, science, and writing measure student progress toward meeting the Sunshine State Standards benchmarks. Norm-referenced tests (NRT) in reading and mathematics compare the achievement of Florida students with that of their peers nationwide.

The FCAT SSS test is based explicitly on the learning goals that Florida educators identified in the Sunshine State Standards and is developed, administered, and scored with the active participation of hundreds of Florida educators and citizens. The FCAT NRT provides information to help ensure that Florida students are keeping pace with their peers nationally. Comparing Florida students to those around the nation requires that the NRT not be too closely aligned with the curriculum of any one state, so the NRT is not necessarily aligned with the Sunshine State Standards. The *Stanford Achievement Test, Tenth Edition®* (*Stanford 10* or *SAT 10*) has been used as the FCAT NRT since 2005. In this volume, the term *FCAT* is used to refer only to the CRT portion, or the FCAT SSS. For more information about the FCAT NRT, refer to the DOE website, http://fcat.fldoe.org/fcatpub2.asp.

Lessons Learned REPORT DEVELOPMENT

FCAT Mathematics Lessons Learned Report Development

Lessons Learned Task Force

For this *Lessons Learned* volume, the DOE analyzed data and identified statewide trends in student mathematics performance based on FCAT Mathematics scores for Grades 3–10 from 2001 through 2005. In September 2006, the DOE convened a task force of Florida educators to review the data analysis from the 2001–2005 FCAT administrations, review test items, generate implications for student instruction, and make observations. The task force included curriculum supervisors, resource teachers, school administrators, and curriculum specialists. For the purpose of data analysis and test item review, the larger task force was divided into elementary, middle, and high school focus groups. All task force members had extensive experience with the Sunshine State Standards, the FCAT, and classroom instruction. The work of the task force included reviewing overall test results, results for strands, and question-level results. DOE staff and the DOE's test development contractor assisted task force members in understanding student performance data and facilitated the production of the report. Additional details of the report development process are outlined in the content section.

The report contains results and implications for instruction derived from the synthesis and analysis of several types of data. The task force received three specific bodies of state-level data: mean developmental scale scores (DSS); the percent of students scoring in Achievement Levels 3, 4, and 5; and the mean percent correct by strand. The data presented in this report are from the FCAT Mathematics test administrations from 2001 through 2005; however, special ad hoc analyses of these data were conducted specifically for *Lessons Learned* and have not been reported previously. These ad hoc analyses incorporate results from all student curriculum groups except Home Education.

The following table illustrates the student populations included in the first *Lessons Learned* report, this *Lessons Learned* report, and the state-level score reports for regular FCAT administrations.

Table 1: Student Populations in *Lessons Learned*			
Student Populations	***Lessons Learned* (2002)**	***Lessons Learned* (2007)**	**FCAT Regular Administration State-Level Results**
Standard Curriculum ELL[1] for more than 2 yrs Nondisabled ESE[2] Gifted Speech Impaired Hospital Homebound	✓	✓	✓
ESE		✓	✓
ELL		✓	✓
Home Education			

[1] English Language Learner (ELL)
[2] Exceptional Student Education (ESE)

Premises

The results contained in this document are based on several important premises that the users of this report should consider carefully.

- The first premise is that this kind of data analysis project could be conducted accurately. The authors presumed if professional Florida educators were provided the opportunity to analyze the FCAT test results and the FCAT test questions, meaningful conclusions related to student learning could be reached. It is believed that the task force's conclusions contained in this document validate this premise.
- The second premise is that Florida educators and others want to know what the FCAT results reveal about education in the state. The authors recognized that classroom teachers are continually seeking to improve student learning and to help students meet the challenging expectations presented in the Sunshine State Standards. The structure and content of *FCAT Mathematics Lessons Learned* is intended to facilitate these processes.
- The third premise concerns the FCAT data. The authors presumed that overall student effort after the field-test year remained constant. That is, students were consistently giving their best performance on the state assessments. Any variations in performance from year to year should not be explained as the result of varied student effort, but instead, as the result of other factors such as decline or improvement in student learning.

- The final premise about the FCAT data is that the content assessed by the test remains stable from year to year. Supporting this presumption, it is important to note that the FCAT consistently measures the same benchmarks of the Sunshine State Standards, even though individual test items vary. Year-to-year comparability at the overall test level is further supported by the use of sophisticated statistical models that account for any variance in test difficulty. For more information about test equating, see page 11.

Structure of *FCAT Mathematics Lessons Learned*

The guiding principle for the development of this publication has been an emphasis on the importance of "teaching to the standards" (the Sunshine State Standards) rather than "teaching to the test" (the FCAT). In support of this principle, *Lessons Learned* is organized using the same structure as the Sunshine State Standards. Results are presented for overall achievement as well as for the strands assessed on the FCAT:

Strand A: *Number Sense, Concepts, and Operations*
Strand B: *Measurement*
Strand C: *Geometry and Spatial Sense*
Strand D: *Algebraic Thinking*
Strand E: *Data Analysis and Probability*

The following table provides the percent distribution of raw score points across mathematics strands, by grade level.

Table 2: Approximate Percent Distribution of Raw Score Points across FCAT Mathematics Content Strands by Grade Level

Grade	Number Sense, Concepts, and Operations	Measurement	Geometry and Spatial Sense	Algebraic Thinking	Data Analysis and Probability
3	30%	20%	17%	15%	18%
4	28%	20%	17%	17%	18%
5–8	20%	20%	20%	20%	20%
9–10	17%	17%	25%	25%	17%

Lessons Learned contains an analysis of statewide trends based on Achievement Levels as well as student performance in each reporting strand by year. A longitudinal comparison of data is also presented. Longitudinal analysis provides comparable data in the sense that there is relatively the same number of students from year to year (minus the movement of students in and out of the state) unless otherwise noted. For example, Grade 3 results in 2001 are compared to Grade 4 results in 2002, which are then compared to Grade 5 results in 2003, and so on.

Observations about students' academic strengths and weaknesses are provided for mathematics strands and for the standards that comprise each strand. This report also presents sample FCAT questions that reflect the kinds of skills students are expected to demonstrate, and provides instructional strategies to help teachers move students toward greater mastery.

Sections in this report are identified with the following icons and appear in the following order:

Introduction and Report Development

Mathematics Data Analysis

Instructional Implications

FCAT Mathematics References

FCAT Resources

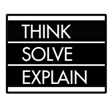
Within each section of this report there are graphical displays of data, interpretations of these data, and implications for instruction. For ease of reference, graphs and tables have been numbered. Graphs are referred to by an abbreviated code; for example, "M-1" refers to the first mathematics graph, titled "Mathematics Grades 3–10 Mean Developmental Scale Scores." The "1" in "M-1" simply refers to the graph order in this publication.

Lessons Learned *contains an analysis of statewide trends based on Achievement Levels as well as student performance in each reporting strand by year.*

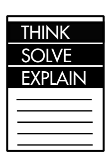
Sample test items (questions) are included throughout this document to help the reader gain as much insight as possible about students' academic strengths and weaknesses. These questions are boxed and presented in distinguishable type. They were selected from several sources, including actual questions from previous test administrations, the *FCAT Mathematics Test Item Specifications*, and the *FCAT Sample Test Materials*. When an actual test question is used as an example, the most current performance statistics available at the time the task force met in September 2006 are presented with the question. These statistics provide additional insights regarding students' academic strengths, weaknesses, and most common mistakes. The correct answer is indicated by a pointing hand symbol (☞). For performance tasks, a top-score response is provided.

The terms *questions* and *items* are used interchangeably in this publication. Incorrect answer options (choices) are also called *distractors*. Item types are abbreviated as follows: MC (multiple-choice question), GR (gridded-response question), SR (short-response question), and ER (extended-response question). In Grades 5, 8, and 10, students are administered MC, GR, SR, and ER questions. Grades 3 and 4 students are administered only MC questions. Grades 6, 7, and 9 students are administered both MC and GR questions. On the FCAT, the icons below help students identify various item types; in this publication, the same icons appear next to GR, SR, and ER items. For detailed information about FCAT Mathematics test items, see the *FCAT Mathematics Test Item Specifications* on the DOE website at http://fcat.fldoe.org/fcatis01.asp.

Mathematics gridded-response (GR) question



Mathematics short-response (SR) question



Mathematics extended-response (ER) question

By examining and reporting the historical results of the FCAT Mathematics test, information about statewide trends in achievement of the Sunshine State Standards as measured by the FCAT can be provided to educators charged with improving students' performance. The objective of this report is to translate this information into insights about student progress within Florida classrooms and schools.

Navigating This Publication

This publication has been designed to include helpful navigation aids for the reader. Each page header provides information about the content discussed on that page, including the chapter title (sometimes abbreviated to fit), the SSS strand label, and, as appropriate, the grade or grade ranges. Page tabs display the grade or the SSS strand letter, and SSS benchmark charts give the full text of the standard and benchmark. The example below identifies each page element.

SSS content strand title:
Number Sense, Concepts, and Operations

Chapter title:
Results by Strand and Instructional Implications

Grade or grade range:
Grades 6–8

SSS content strand letter:
Strand A

SSS standard and benchmark chart:
This chart provides text for Standard A1 and Benchmarks MA.A.1.3.1, MA.A.1.3.2, MA.A.1.3.3, and MA.A.1.3.4.

Lessons Learned STATISTICAL CONSIDERATIONS

Statistical Considerations and the FCAT

Data Analysis Process

The task force analyzed results from 2001 through 2005 to identify trends in student performance. By reviewing results at different levels, the task force identified some areas of growth as well as other areas needing further attention. Question-by-question analyses revealed the extent to which changes in student performance reflected gains in skills associated with mathematics. Task force members were reminded that changes in performance at the strand level may be attributed to variations in the difficulty level of questions from one year to another. As stated earlier, overall changes in difficulty are accounted for with a statistical technique known as equating;[1] however, at the strand level, there may be some variation in difficulty.

Student results are provided in different ways and at different content levels: average developmental scale scores (DSS) at the test level, percent of students who achieve at a proficient level or above (Levels 3, 4, or 5) for the entire test, and mean percent of questions answered correctly at the strand level.

For the second metric listed above, data are reported for students who scored in Levels 3, 4, and 5. For example, if 30% of students scored in Level 3, 20% in Level 4, and 10% in Level 5, the reported percent would be 60%. This implies that the percent of students who did not meet the minimal acceptable level of achievement (Level 3) was 40% (i.e., 100% minus 60%).

[1] Test equating is the process by which scores from two administrations are made comparable—that is, placed on the same scale (the FCAT 100–500 scale). Students tested in two different years take tests that have different questions; however, the tests have a common set of questions called anchor questions. The data used in the statistical equating procedures are gathered from student results on the anchor questions from year to year.

Limitations

Analysis of the mathematics student performance data was limited by a number of conditions. Where salient to the findings of this report, these limitations are noted below.

- The task force evaluated the questions that were particularly challenging to students, identifying persistent areas of concern without attempting to evaluate student results on every question.
- The analyses reported herein are based on state-level data and are not intended to provide specific classroom, school, or district interpretations; however, this presentation of findings may provide a model that can guide data analysis at those levels. The results may also be used by teachers to compare their students' mathematics performance to the average performance of students across the state.
- FCAT DSS are derived from equated scores. Results based on these DSS can be compared across years. These results are the average (mean) DSS and the percents of students in Achievement Levels 3, 4, and 5. Results not based on equated scores include performance tasks at the strand and question level. The task force took care not to misinterpret or overinterpret the trend data presented at these levels, given the limitations with comparability.
- Results are reported at the strand level only, not for the standards that comprise each strand. In mathematics, subscores are reported for the five strands. The task force provided instructional implications at the standard level, based on examination of items within the standards. When FCAT questions are selected each year, the most important consideration is content representation. Consistent content is maintained from year to year by selecting questions for the various strands and benchmarks assessed. Each year's test includes both previously used anchor questions and new questions. During the process of assembling the test, question and test statistical characteristics are compared for the total test, the anchor questions, and the strands. Using these pre-equating methods, similar characteristics are maintained for the tests and strands from year to year. Test equating, conducted during the test scoring process, is used to generate the total test scores.
- The statistic used to report performance by strand is called the mean percent correct. The mean percent correct is calculated in a manner analogous to finding the percent of points a student earned on the questions within each strand. It is the mean number of points earned by the entire group of students divided by the number of points possible. For example, if there are 12 questions within a strand and five students correctly answer 8, 6, 6, 9, and 12 questions, the average number of questions answered correctly is 8.2. This translates to an overall mean percent of 68% (i.e., 8.2 divided by 12).
- For some standards or benchmarks, the reader may note an absence of observations and/or implications for instruction. In these instances, analysis of student performance data available did not yield enough consistent information from which to reach clear conclusions.
- The longitudinal comparisons of student performance data are NOT for a matched cohort of students (i.e., not matched at the individual student level over time). For example, these data simply represent all students tested as third graders in 2001 and all students tested as fourth graders in 2002.

Effect Size and FCAT Results

Effect size is a statistical method that is often used in research. This statistic quantifies change relative to the spread of the distribution. For example, while a difference of two units may be statistically significant with large sample sizes, it may mean very little with regard to the practical size of that difference. The effect size statistic captures the fact that this difference may, from a practical standpoint, be quite small. The information below provides context on how effect size can be used to evaluate FCAT results.

Effect size is a statistical method that . . . quantifies change relative to the spread of the distribution.

The following text identifies the categories of change most often used when describing variance in effect size statistics.[2]

Effect Size	Qualifier
$d < 0.2$	Negligible
$0.5 > d \geq 0.2$	Small
$0.8 > d \geq 0.5$	Medium
$d \geq 0.8$	Large

Although the effect size can be computed as a negative value, its interpretation is based on its absolute value; therefore, a negative effect size should not be interpreted as a negligible change. A small effect size implies a change that is insubstantial. For example, if the average DSS increases by 100 points, the effect size is 0.33 (100 divided by 300, assuming 300 is a pooled standard deviation). While this change may be statistically significant because of the large number of student scores, it is small with regard to effect size. By contrast, an increase of 600 developmental scale score points would yield an effect size of 2.0. This would be a large effect size, indicating that relative to the standard deviation of the DSS (300), there was a substantial change in the average DSS. From a program evaluation standpoint, the implications of medium or large effect sizes warrant more serious consideration than small effect sizes. As shown in the Grade 10 row of Table 3 on the following page, there was relatively no change (the effect size was 0.0) in mean DSS from 2001 through 2005. A slight difference or change in score is equivalent to a negligible effect size or no effect size.

It should be noted that the effect size qualifiers suggested by Cohen are general and that many researchers advocate thoughtful consideration of the evaluated content. For example, a "medium" effect size change may be considered large in a social science like education. For the purposes of this report, the general parameters proposed by Cohen will be utilized. Further research and consideration will be given to interpretation of effect sizes for incorporation into future *Lessons Learned* reports.

[2] Cohen, J. (1988). *Statistical power analysis for the behavioral sciences.* Hillsdale, NJ: Erlbaum.

Technically, effect size is the ratio of change to the standard deviation. For this report, a pooled standard deviation was used.[3] The pooled standard deviation is used typically when there are two samples being compared with varying *N*-counts. The effect size ratio can vary from 0 (no change) to infinity.

$$\text{Effect Size} = d = \frac{\bar{\mu}_{new} - \bar{\mu}_{old}}{\sqrt{(N_{new}\sigma^2_{new} + N_{old}\sigma^2_{old})/(N_{new} + N_{old})}}$$

In this equation, $\bar{\mu}_{new}$ represents the mean of the new DSS, $\bar{\mu}_{old}$ is the mean of the old DSS, N is the sample size, and σ^2 is the squared standard deviation.[4] The following table provides the statistics used to calculate effect sizes.

Table 3: Statistics Used to Calculate Effect Sizes

Grade	2001			2005			Difference in Means	Effect Size
	Sample Size	Mean	Standard Deviation	Sample Size	Mean	Standard Deviation		
3	186,472	1260	295	203,156	1380	312	120	0.4
4	188,933	1394	283	195,990	1509	253	115	0.4
5	187,794	1580	298	181,545	1648	234	68	0.3
6	187,283	1592	295	201,760	1653	266	61	0.2
7	183,382	1724	275	202,581	1778	247	54	0.2
8	174,561	1847	229	201,783	1866	219	19	0.1
9	191,246	1864	218	214,534	1918	198	54	0.3
10	144,837	1976	196	178,720	1978	193	2	0.0

The data provided in Table 3 indicate that there was, for the most part, small effect size change when comparing DSS from 2001 to 2005. With the exception of Grades 8 and 10, the effect size ranged from 0.2 to 0.4. In Grades 8 and 10, though, growth was negligible (0.1 and 0.0 effect sizes, respectively). These findings, along with other results that are presented in *Lessons Learned*, are intended to provide another measure of program and student performance for educators to consider and use to evaluate the efforts of Florida educators and students as a whole.

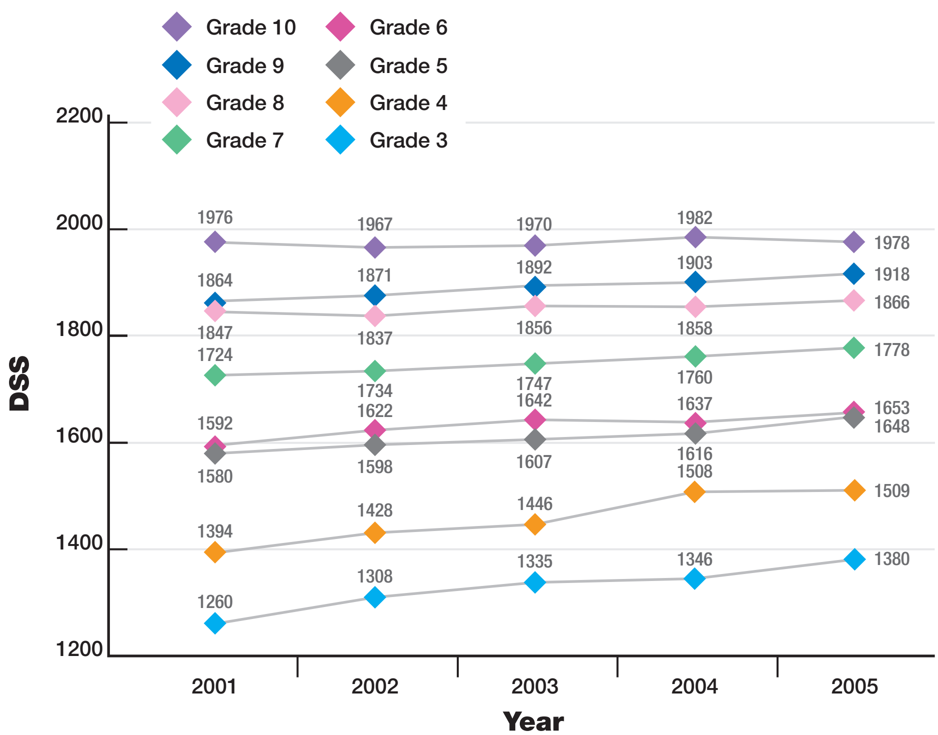
[3] Yen, W.M. (1986). The choice of scale for educational measurement: An IRT perspective. *Journal of Educational Measurement,* 23, 299–325.

[4] Ibid.

Mean Developmental Scale Scores (DSS)

A beginning point for analyzing mathematics performance trends is reviewing the statewide mean DSS from 2001 through 2005. In its review of these data, the task force examined the mean DSS for all curriculum students in Grades 3–10. The following graph shows a summary of the mean developmental scale scores.

Graph M-1
Mathematics Grades 3–10
Mean Developmental Scale Scores



As shown in the graph, mean DSS results for Grades 3, 4, and 5 have increased steadily from 2001 through 2005. Grade 3 results have increased by 120 DSS points, moving from 1260 to 1380. This is a small increase with regard to effect size (0.4). Grade 4 results increased by 115 points (a small effect size of 0.4), moving from 1394 to 1509. Results for Grade 5 students increased by 68 points, beginning at 1580 and increasing to 1648, also a small effect size of 0.3.

The increases in mean DSS were less pronounced for the middle school grades. Grade 6 results increased by 61 points, moving from 1592 to 1653 over the five-year period. The five-year increase for Grade 7 was 54 points, increasing from 1724 to 1778. Increases at both Grade 6 and Grade 7 were negligible with regard to effect size (0.2 for both). Grade 8 results remained about the same over the five-year period, increasing by 19 points from 2001 through 2005. The effect size of 0.1 indicates a negligible change.

Grade 9 results increased by 54 points from 2001 through 2005, a small effect size of 0.3. The trend was flat for Grade 10, where the mean DSS showed little variation and increased by two points over the five-year period, representing an insubstantial effect size.

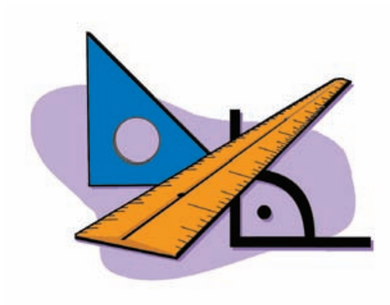
It is important to note that the equivalent of one year's growth on the FCAT developmental scale decreases as students move up in grade level. For example, the equivalent of one year's growth for Grade 4 students is an increase of 162 DSS points. This score increase, representing a year's growth, diminishes steadily to 48 for Grade 10 students. The reader is cautioned to consider this fact while reviewing these results. The following table provides growth expectations for all grades.

Table 4: One Year's Growth Definition for FCAT Mathematics Developmental Scale Scores

Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Grade 9	Grade 10
162	119	95	78	64	54	48

While these results provide valuable information with regard to overall student achievement, they do not provide educators with information that allows them to more effectively target their time and resources toward *specific* areas of need. The following sections provide data and analyses at the grade and strand level, along with instructional strategies suggested by the task force at the standard level.

It is important to note that the equivalent of one year's growth decreases as students move up in grade level.



Lessons Learned
RESULTS BY GRADE LEVEL

FCAT Mathematics Statewide Achievement Results by Grade Level

Achievement Levels and DSS

Performance on the FCAT Mathematics test is reported by Achievement Level as well as by DSS. The Achievement Levels for each grade were recommended by teachers and district administrators in 1998 and adopted by the Florida Board of Education. Five levels are used to divide the DSS range into categories of achievement for each grade. A brief description of the five categories follows.

- **Level 5**—This student has success with the most challenging content of the Sunshine State Standards. A student scoring in Level 5 answers most of the test questions correctly, including the most challenging questions.
- **Level 4**—This student has success with the challenging content of the Sunshine State Standards. A student scoring in Level 4 answers most of the test questions correctly, but may have only some success with questions that reflect the most challenging content.
- **Level 3**—This student has partial success with the challenging content of the Sunshine State Standards, but performance is inconsistent. A student scoring in Level 3 answers many of the test questions correctly, but is generally less successful with questions that are the most challenging.
- **Level 2**—This student has limited success with the challenging content of the Sunshine State Standards.
- **Level 1**—This student has little success with the challenging content of the Sunshine State Standards.

The following table provides the ranges for each Achievement Level by grade.

Table 5: Achievement Levels in FCAT Mathematics Developmental Scale Scores					
Grade	Achievement Level				
	Level 1	Level 2	Level 3	Level 4	Level 5
3	375–1078	1079–1268	1269–1508	1509–1749	1750–2225
4	581–1276	1277–1443	1444–1657	1658–1862	1863–2330
5	569–1451	1452–1631	1362–1768	1769–1956	1957–2456
6	770–1553	1554–1691	1692–1859	1860–2018	2019–2492
7	958–1660	1661–1785	1786–1938	1939–2079	2080–2572
8	1025–1732	1733–1850	1851–1997	1998–2091	2092–2605
9	1238–1781	1782–1900	1901–2022	2023–2141	2142–2596
10	1068–1831	1832–1946	1947–2049	2050–2192	2193–2709

These Achievement Levels were adopted in 1998, prior to the establishment of No Child Left Behind (NCLB) in 2002. NCLB requires states to identify one of their Achievement Levels as "proficient." The "partial success" FCAT Mathematics Achievement Level (Level 3) was identified by the State of Florida to be the equivalent of what the federal legislation would deem "proficient."

The grade-specific sections that follow include graphic information organized by the following categories:

- Achievement Level 3 (proficient or partial success) or higher (e.g., Graph M-2)—Following NCLB guidelines, students who demonstrate partial success on the content assessed by the FCAT have DSS classified as Level 3 achievement; therefore, Level 3 will be used to divide these DSS into two groups: Levels 1–2 and Levels 3–5.
- Five-year trend results for Achievement Levels 1 through 5 (e.g., Graphs M-3 and M-4)—Evaluating results at this level allowed the task force to better understand the changes that took place related to score distribution across Achievement Levels over time.
- The five Sunshine State Standards strands described in the introduction (e.g., Graph M-5)—Since 2001, FCAT Mathematics reports sent to students, parents, and teachers have provided the number of questions correct by strand. The ad hoc analyses by strand were conducted specifically for this report and are based on the mean percent correct statistic. The focus of this section will be on results across strands by grade.

Note: At the state level, the statistical process of equating allows for the across-year comparison of the mean percent of students who achieve in Level 3 or higher; however, it is not appropriate to draw trend-related inferences with mean percent correct statistics across years and within a given strand.

Given educators' desire to glean reliable information from any test that is administered to their students (including the FCAT), it is important to identify the strand-level comparisons that yield valid interpretations of student performance. While the comparisons that are described in the following paragraphs could not be used in this *Lessons Learned* report (a state-level report), they can be applied in school- and district-level evaluations. The state data in Tables 6 and 7 below are real; however, the school and district data are not. For illustration purposes, mock data are provided for the fictitious schools and district in the tables.

One valid comparison is performance on a given strand between schools, districts, and the state. For example, a school's strand-level results can be compared to other schools', districts', or the state strand-level results. District results can be compared to other district results and state results. The reasoning for this is simple: students in any group (school, district, or state) will take the same set of test items in a given year. This means that, regardless of varying item difficulty at the strand level, students are assessed using the same items; subsequently, their results are comparable.

In Table 6 below, students in two schools (Sunshine and Evergreen) and students in the district (Coastal) can be compared to students in the state, based on their performance on Strand A.

Table 6: Mean Percent Correct for Grade 3 Math, Strand A
2005 School Year (mock data)

Sunshine Elementary (mock data)	Evergreen Elementary (mock data)	Coastal District (mock data)	State of Florida (real data)
48%	55%	59%	57%

Another type of valid comparison is the trend of any of the aforementioned comparisons (e.g., school to school, school to district). For example, educators in a low-performing school may be interested in tracking the gap between their students' performance on Strand A, students' performance in their district, and students' performance in the state. Evaluating trend data for such a comparison is valid and potentially very enlightening.

Table 7: Mean Percent Correct for Grade 3 Math, Strand A
2001 through 2005 (mock data)

Year	Sunshine Elementary (mock data)	State of Florida (real data)	Difference
2001	37%	61%	−24%
2002	33%	55%	−22%
2003	36%	55%	−19%
2004	38%	59%	−21%
2005	48%	57%	−9%

In Table 7, the trend results from 2001 to 2005 provide important evaluative information to the educators in Sunshine Elementary. While student performance in Sunshine Elementary was consistently lower on Strand A than the performance of all Grade 3 students in Florida, the progress that has been made over the five-year period is substantial enough to warrant another look at program initiatives (e.g., the school may have introduced an afterschool tutoring program that can be linked to an improvement in performance).

Table 8: Mean Percent Correct for Grade 3 Math, 2005 School Year
Comparison of School to District and School to State (mock data)

Strand	Evergreen Elementary (mock data)	Coastal District (mock data)	Difference	Evergreen Elementary (mock data)	State of Florida (real data)	Difference
Strand A	55%	59%	−4%	55%	57%	−2%
Strand B	70%	61%	9%	70%	68%	2%
Strand C	65%	67%	−2%	65%	60%	5%
Strand D	54%	57%	−3%	54%	59%	−5%
Strand E	52%	65%	−13%	52%	66%	−14%

In Table 8, 2005 mock results for Evergreen Elementary are compared to both the district (Coastal) and the state. This presentation of data provides yet another perspective of student performance and program effectiveness. For example, in Strand B, Evergreen Elementary had a higher mean percent correct statistic than the Coastal District (70% versus 61%, respectively); however, Evergreen Elementary results were comparable to the state (70% versus 68%, respectively). If this variance was consistent over the five years, there would be good reason to identify and share best practices in Evergreen Elementary with the rest of the district.

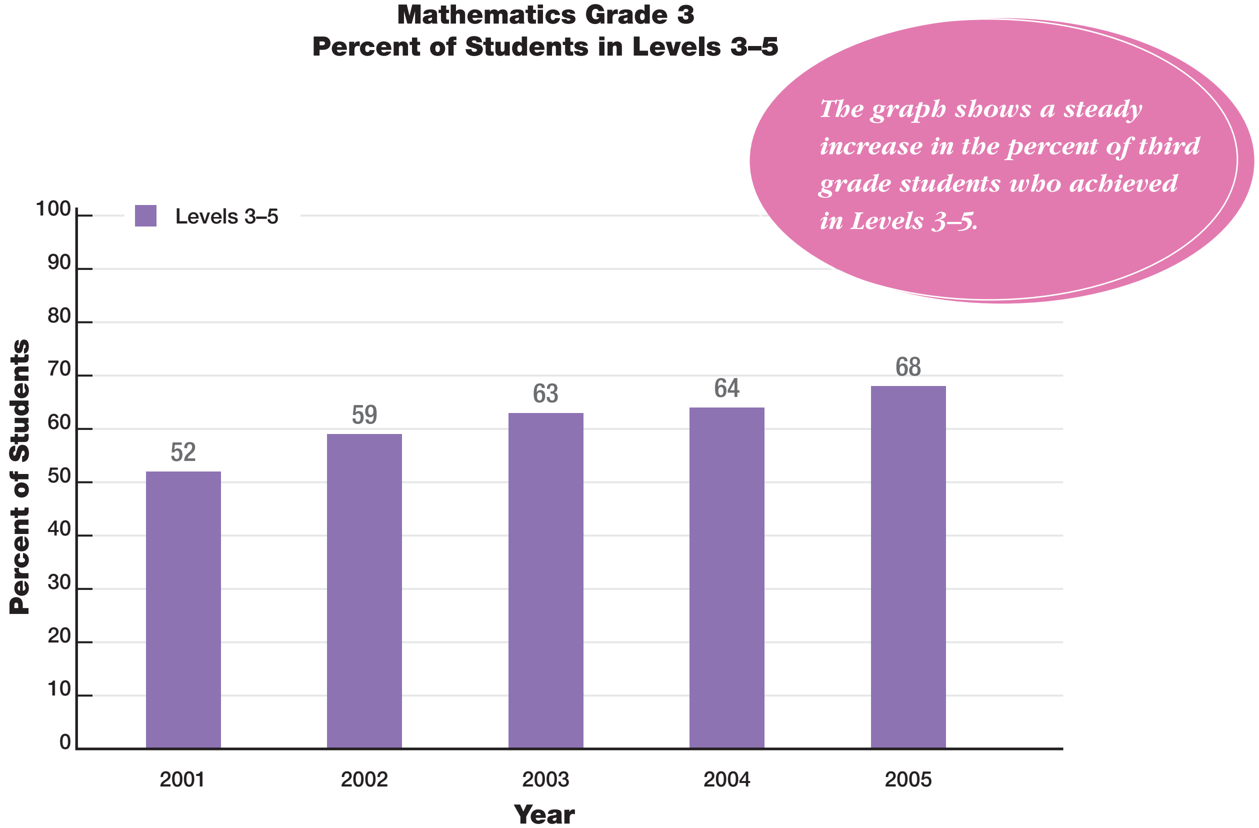
Another meaningful finding from Table 8 is illustrated in Strand C results. In this strand, Evergreen Elementary had a slightly lower mean percent correct than Coastal District (65% versus 67%, respectively); however, this same statistic was higher than that of the state (65% versus 60%, respectively). It would be easy to miss the fact that, while Evergreen Elementary's performance on Strand C was lower than that of the district, the performances of both were substantially higher than the state. Subsequently, it is possible that targeting additional resources to improve performance in Strand C would be a lower priority.

The DOE and the task force encourage educators to use FCAT results in any way that is statistically appropriate. The comparisons that have been described in this section provide possibilities for evaluation at the school and district levels.

Analysis of FCAT Mathematics Grade 3 Results 2001–2005

The results in the following graph show the percent of Grade 3 students who achieved in Level 3 or higher on the FCAT from 2001 through 2005.

Graph M-2
Mathematics Grade 3
Percent of Students in Levels 3–5

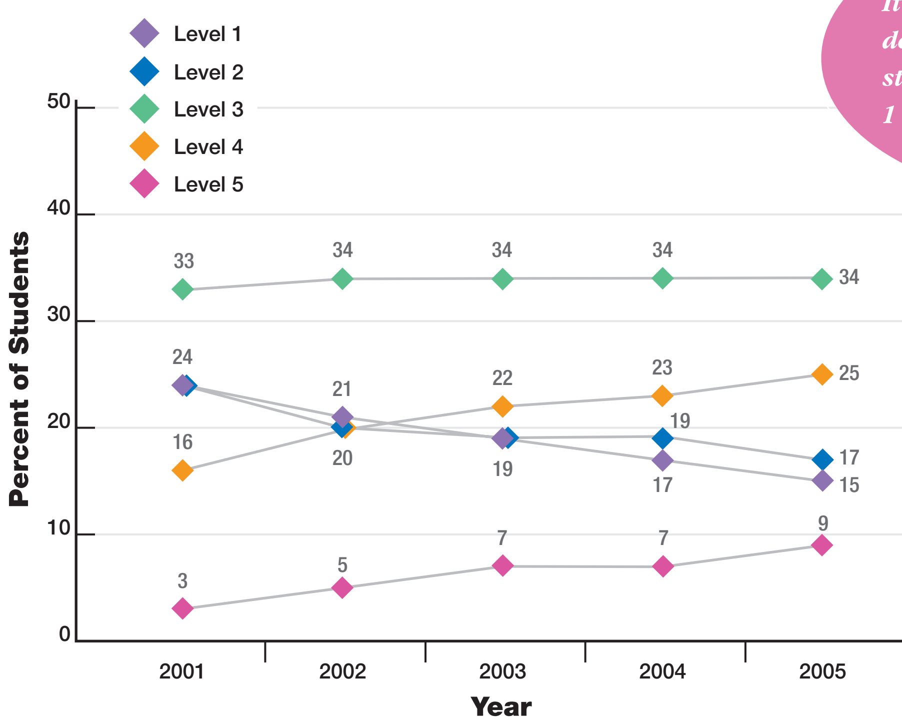


The graph shows a steady increase in the percent of third grade students who achieved in Levels 3–5. The increase from 2001 through 2005 was 16 percentage points. The graph also shows that the percent of students who achieved in Levels 3–5 was 68% in 2005.

Grade 3

It is also worthwhile to study trends for each specific Achievement Level. In the following graph and similar graphs that follow, a positive trend is indicated by a steady decrease in the percents of students achieving in Levels 1 and 2 and by a commensurate percent increase in Levels 3, 4, and 5.

Graph M-3
Mathematics Grade 3
Percent of Students by Achievement Level



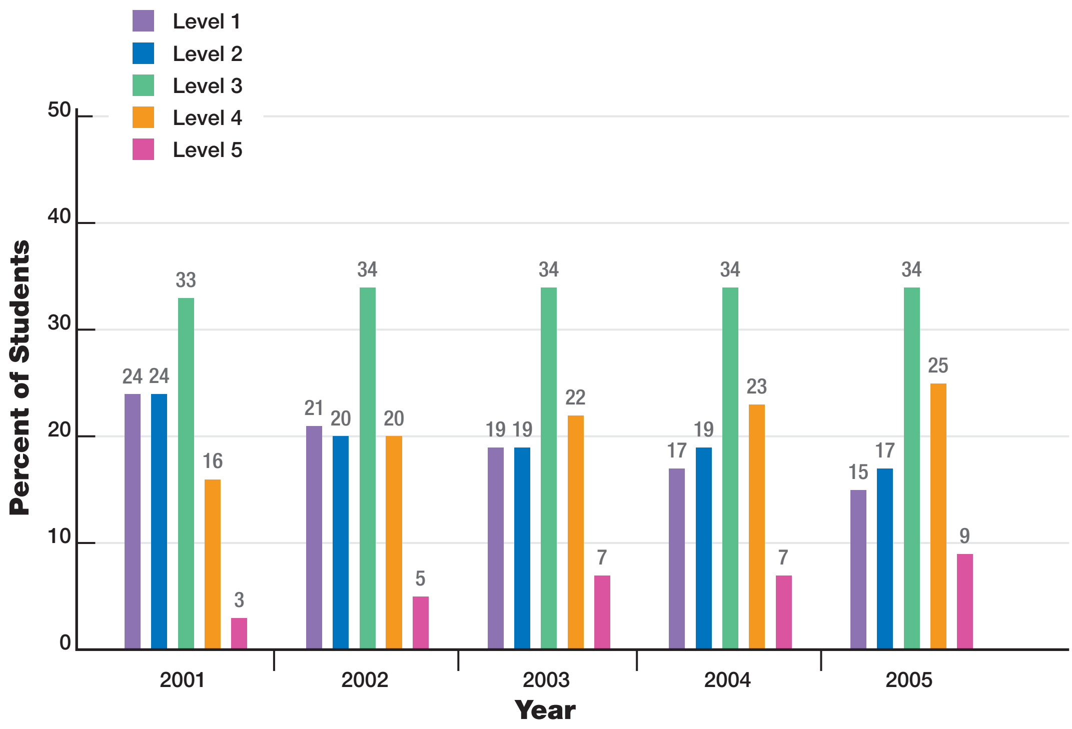
It is encouraging to see a decrease in the percents of students who achieved in Levels 1 and 2.

For Grade 3, it is encouraging to see a decrease in the percents of students who achieved in Levels 1 and 2 (i.e., there should be fewer students in Levels 1 and 2, and more students in Levels 3, 4, and 5). In 2001, the percent of students with scores in each of Levels 1 and 2 was 24%. The percent dropped to 15% and 17% in 2005 for Level 1 and Level 2, respectively. Students performed better overall, as shown by the increase in Levels 4 and 5 percents over the five-year period. Notice that the percents for Level 3 were at 33% or 34% over the five-year period, and that the percents for Levels 4 and 5 increased by 9 and 6 percentage points, respectively.

Grade 3

The information presented in Graph M-3 can also be illustrated with a bar graph. Doing so provides a perspective that captures the distribution within each year, whereas the previous graph more clearly illustrates Achievement Level results over time. Graph M-4 shows a noticeable distributional shift to Levels 4 and 5.

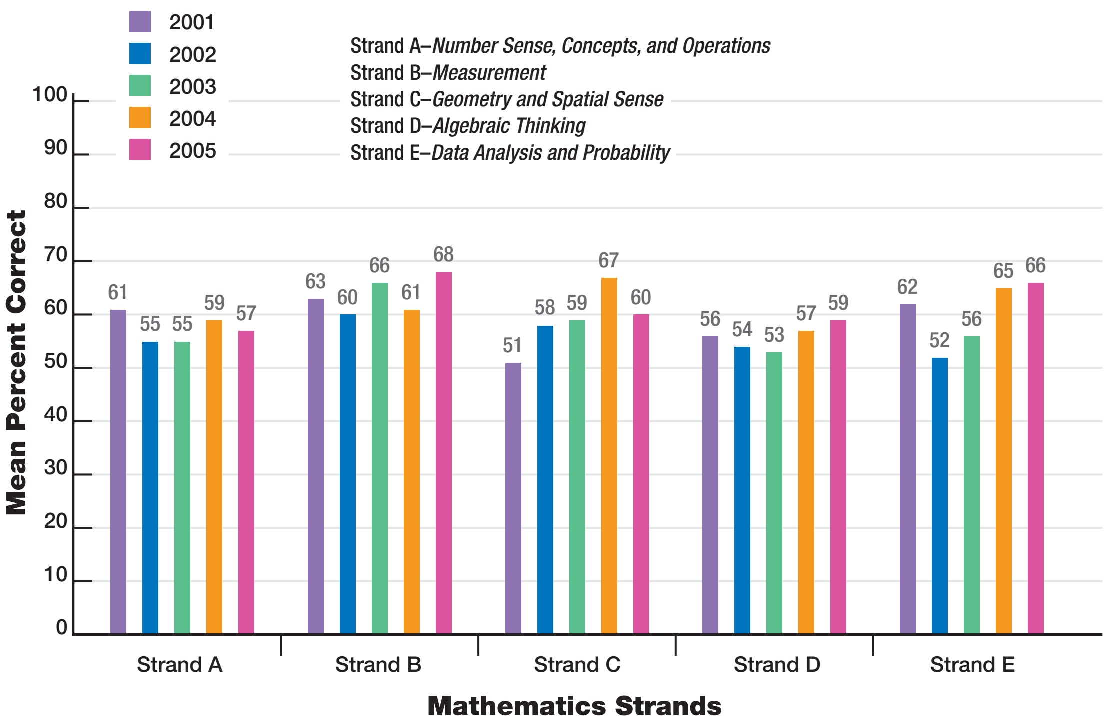
Graph M-4
Mathematics Grade 3
Percent of Students by Achievement Level



Grade 3

Grade 3 student achievement in each strand is provided in the following graph. Readers should pay particular attention to the overall performance across strands. While questions across administrations and within a strand are similar in the content to which they align, the results at the strand level are not equated. Through whole test equating, scores across years are kept on the same scale, despite some variability of test questions or strand difficulty. Any changes in average question difficulty are not adjusted at the strand reporting level; therefore, it is important to realize that the changing results across administrations may reflect, in part, variance in question difficulty from year to year.

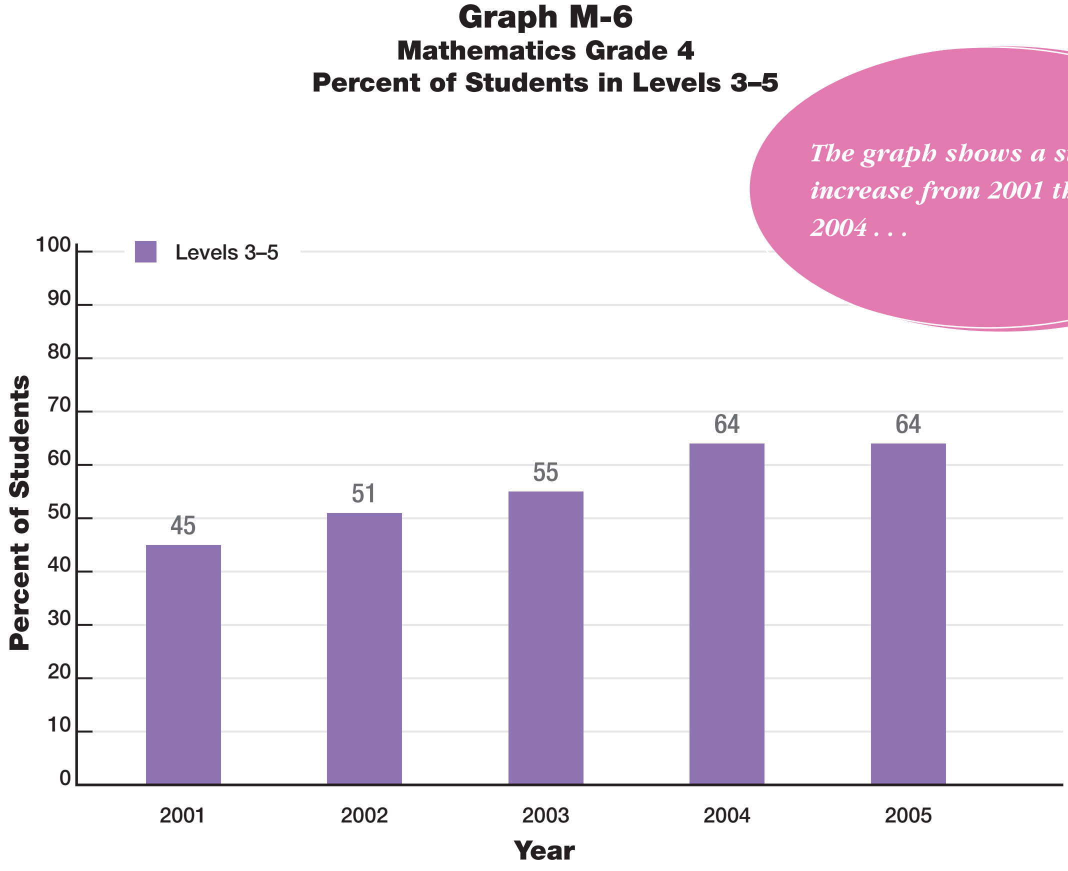
Graph M-5
Mathematics Grade 3
Mean Percent Correct by Strand



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

Analysis of FCAT Mathematics Grade 4 Results 2001–2005

The results for the percent of Grade 4 students who achieved in Level 3 or higher are provided in the following graph.

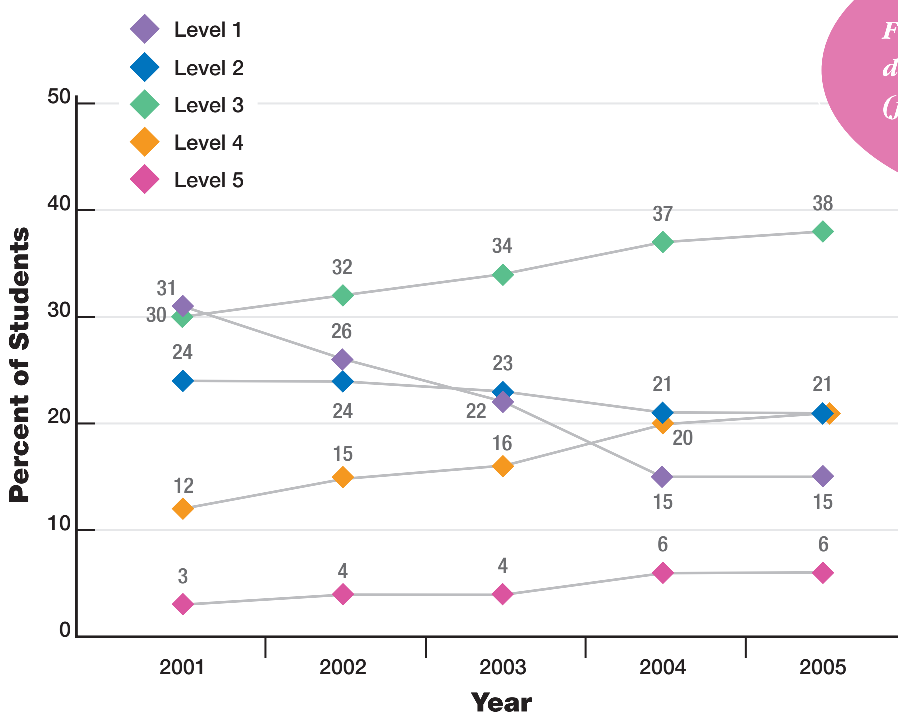


The graph shows a steady increase from 2001 through 2004 . . .

The graph shows a steady increase from 2001 through 2004, and a leveling off in student performance from 2004 through 2005. The increase from 2001 through 2004 was 19 points. Most recent results show the percent of students who achieved in Level 3 or higher at 64%.

The following graph provides a detailed view of the shift in Grade 4 Achievement Levels over the five-year period.

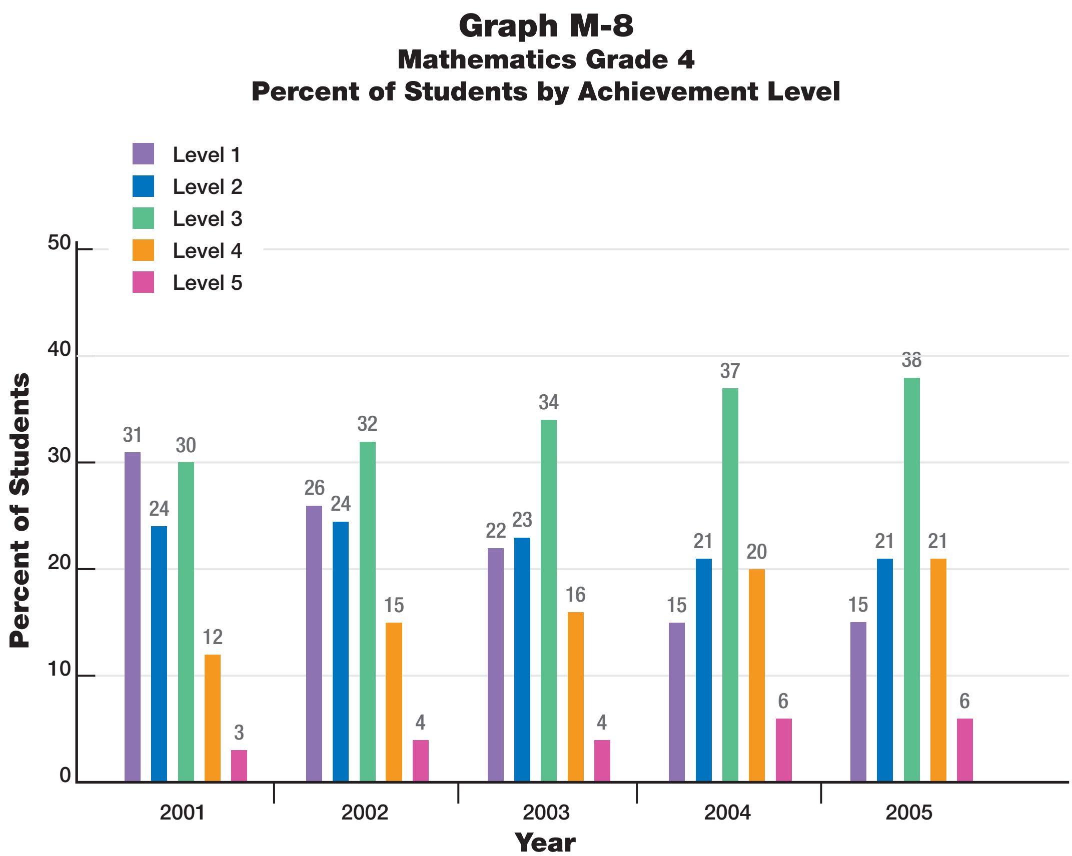
Graph M-7
Mathematics Grade 4
Percent of Students by Achievement Level



For Grade 4, there was a decrease of 16 percentage points (from 31% to 15%) in Level 1.

For Grade 4, there was a decrease of 16 percentage points (from 31% to 15%) in Level 1. While still favorable in the sense that there was a decreasing trend, the drop was not as substantial in Level 2 (3 percentage points). The decrease in Levels 1 and 2 was counterbalanced by an increase in Levels 3 and 4. The increase of 3 percentage points in Level 5 was less pronounced.

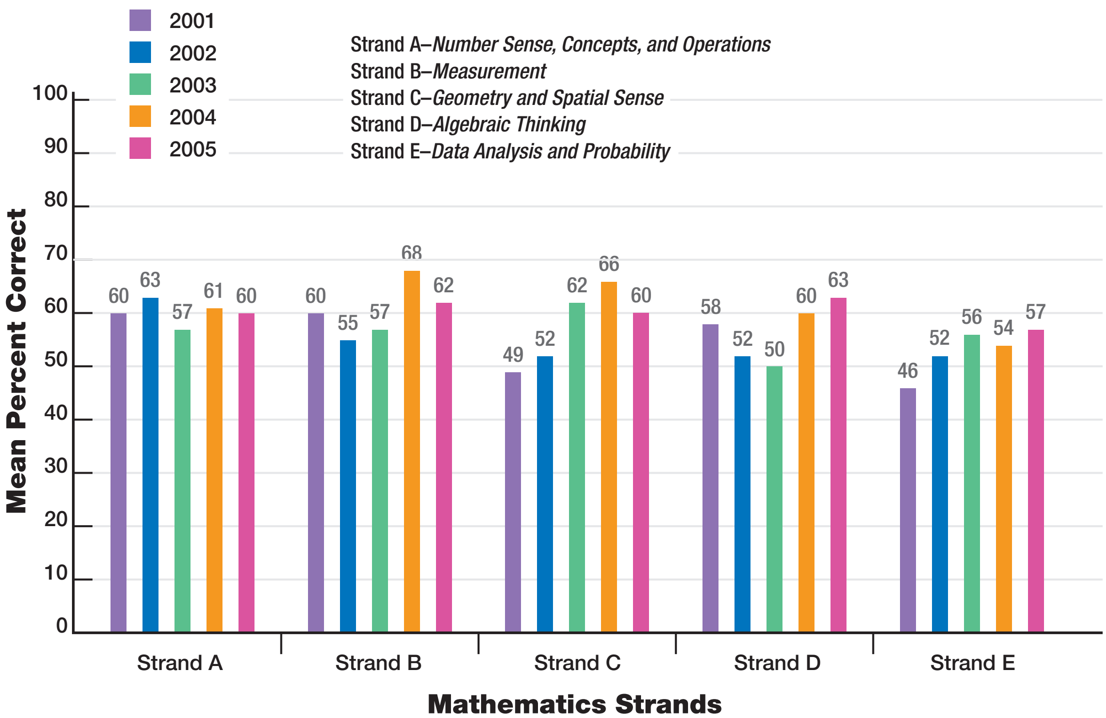
A bar graph displaying the same data as M-7 is provided below. The graph clearly illustrates the shift in Grade 4 from Achievement Levels 1 and 2 to Achievement Levels 3 and 4.



Grade 4

The following graph provides a view of Grade 4 student achievement at the strand level.

Graph M-9
Mathematics Grade 4
Mean Percent Correct by Strand



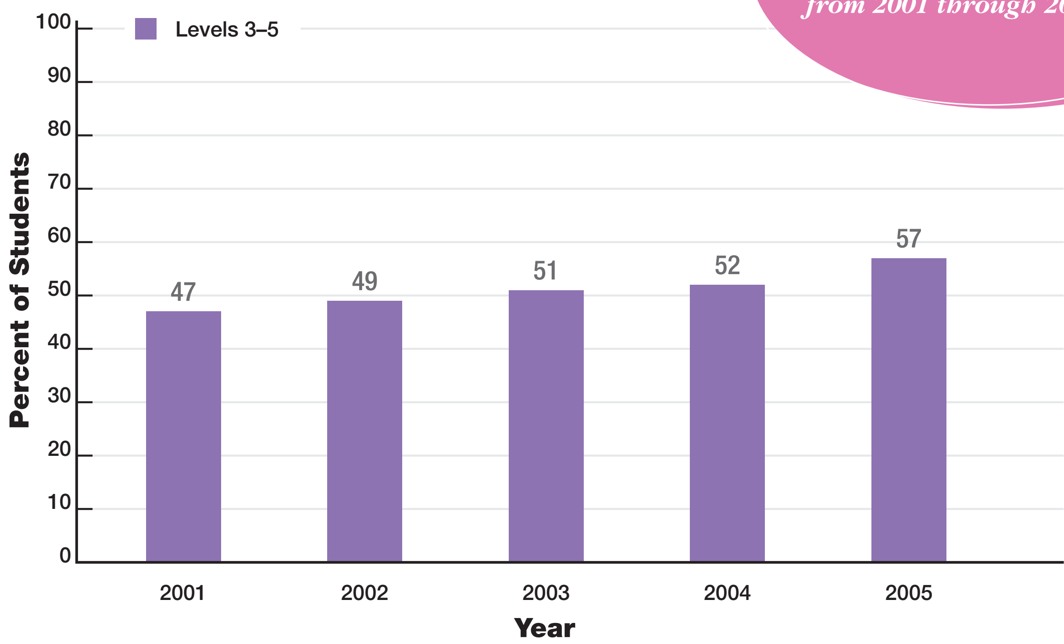
Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

Analysis of FCAT Mathematics Grade 5 Results 2001–2005

The results for the percent of students who achieved in Level 3 or higher in Grade 5 are provided in the following graph.

Graph M-10
Mathematics Grade 5
Percent of Students in Levels 3–5

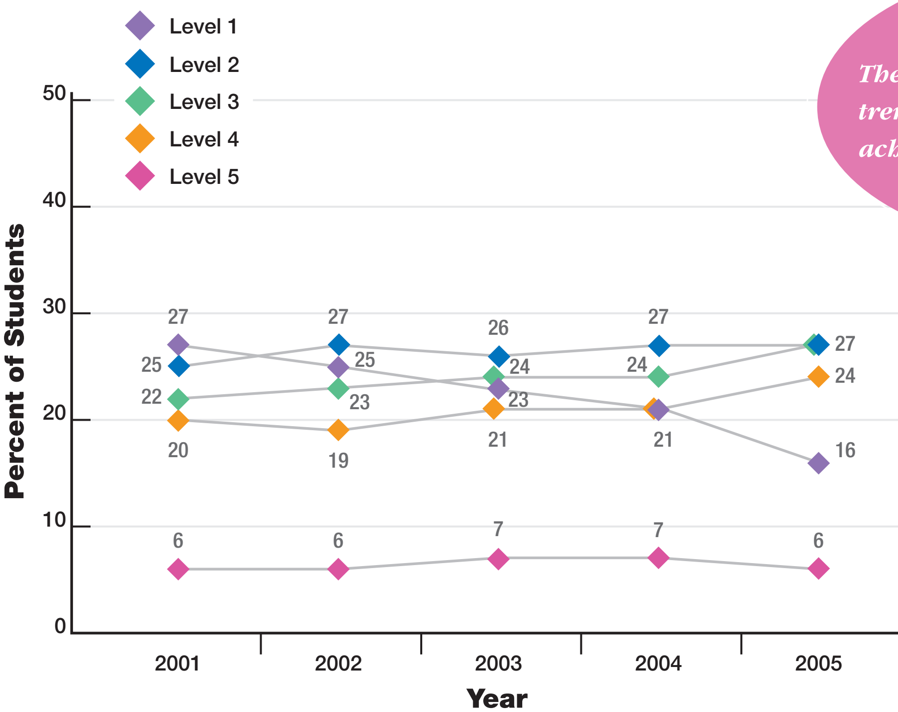
The graph shows a steady increase of 10 percentage points from 2001 through 2005.



The graph shows a steady increase of 10 percentage points from 2001 through 2005. The largest increase took place in 2005 when there was a 5-percentage-point improvement from 2004. The 2005 results show that the percent of students who achieved in Level 3 or higher was 57%.

Trends for Grade 5 over the five-year period are provided in the following graph.

Graph M-11
Mathematics Grade 5
Percent of Students by Achievement Level

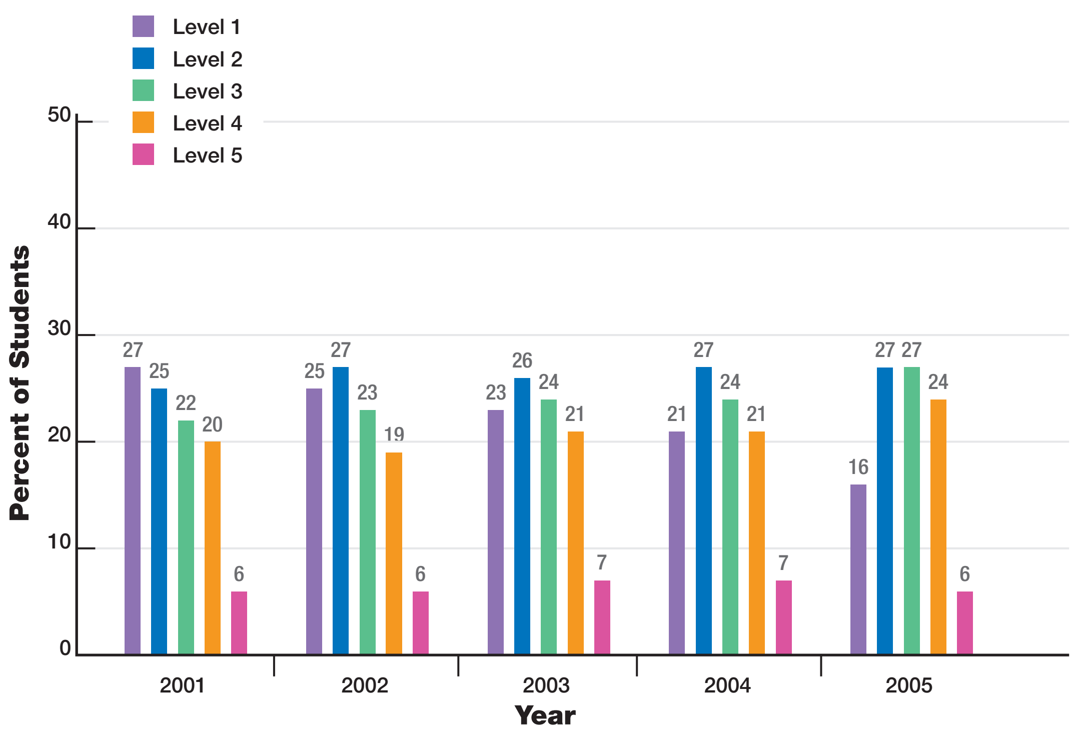


There was a favorable downward trend in the percent of students achieving in Level 1 . . .

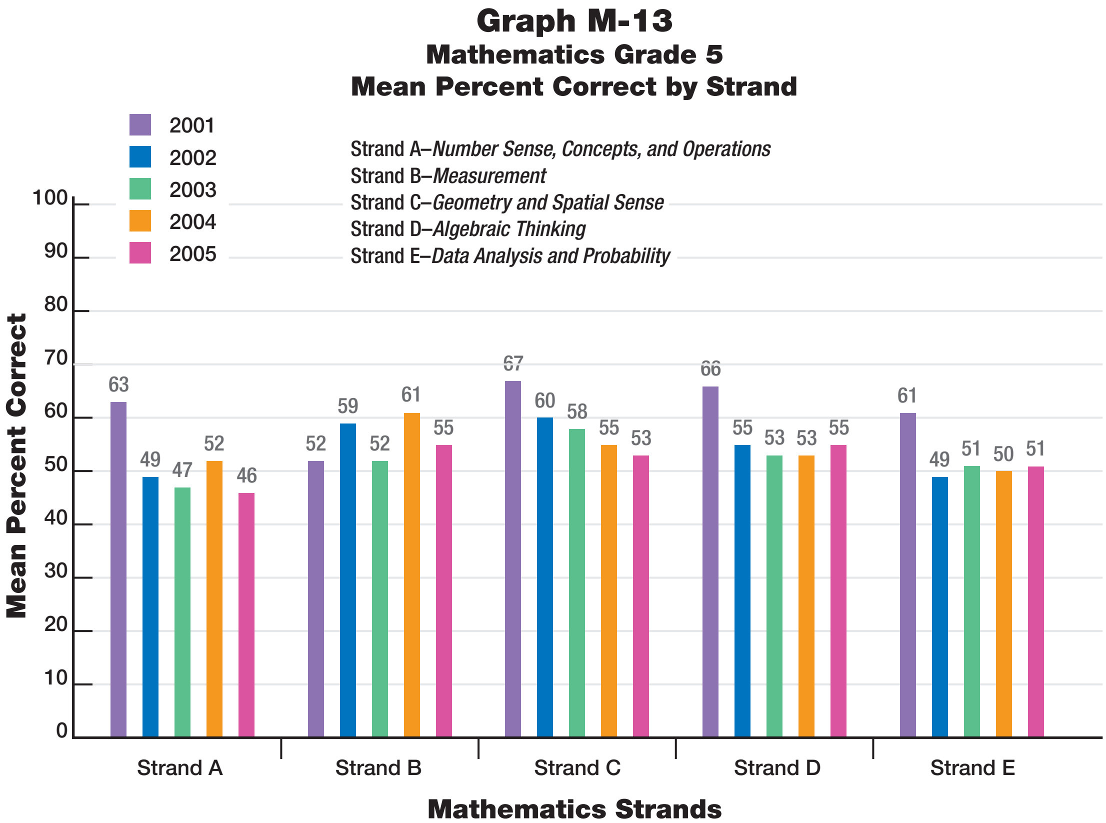
There was a favorable downward trend in the percent of students achieving in Level 1, where the percent from 2001 through 2005 fell by 11 percentage points. The trend for students achieving in Level 2 remained roughly the same over the five-year period. From 2004 through 2005, there was an increase in the percent of students achieving in Levels 3 and 4. Prior to 2004, the trend lines were flat. For Level 5 achievement, the percent varied between 6% and 7%, showing neither a positive nor a negative trend.

The following bar graph highlights the distributional shift from year to year away from Level 1 and toward higher Achievement Levels in Grade 5. Most of the change took place in 2005.

Graph M-12
Mathematics Grade 5
Percent of Students by Achievement Level



The following graph provides results for Grade 5 students at the strand level.

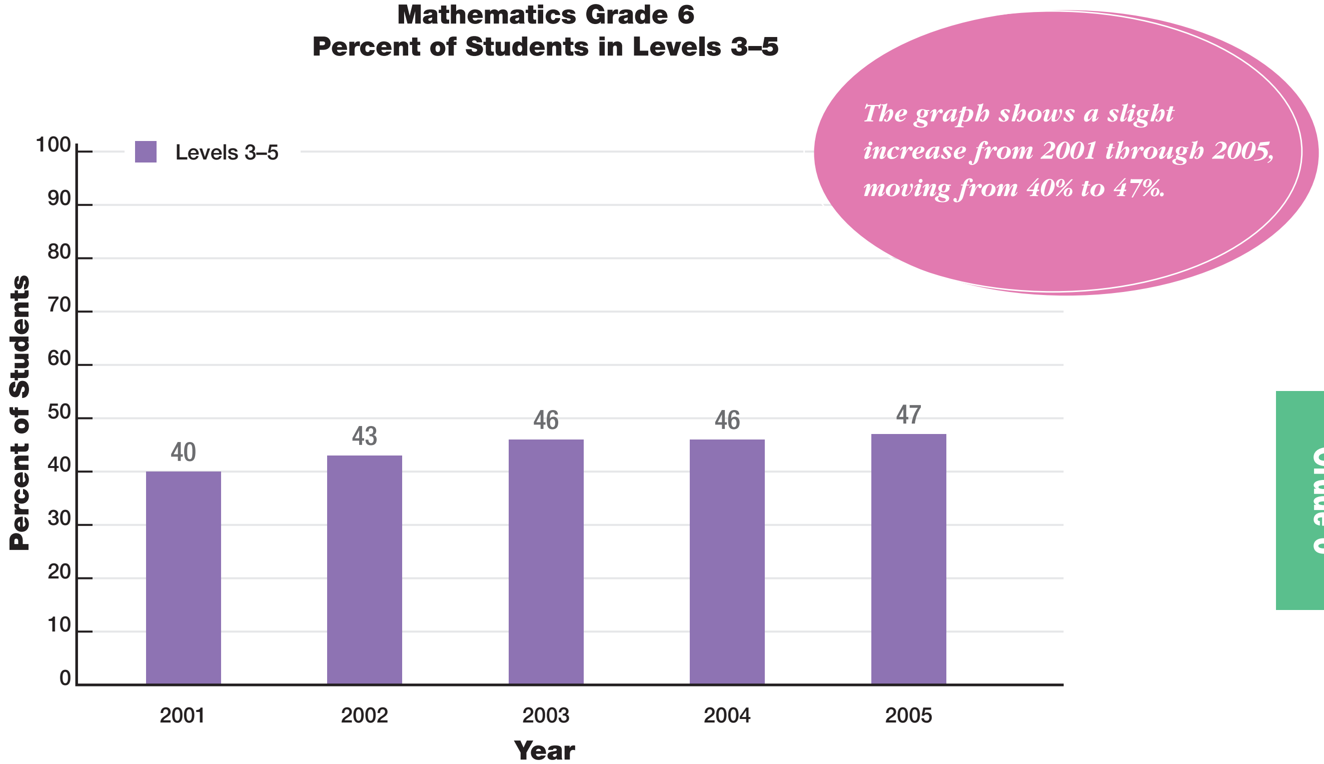


Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

Analysis of FCAT Mathematics Grade 6 Results 2001–2005

The results for the percent of students who achieved in Level 3 or higher in Grade 6 are provided in the following graph.

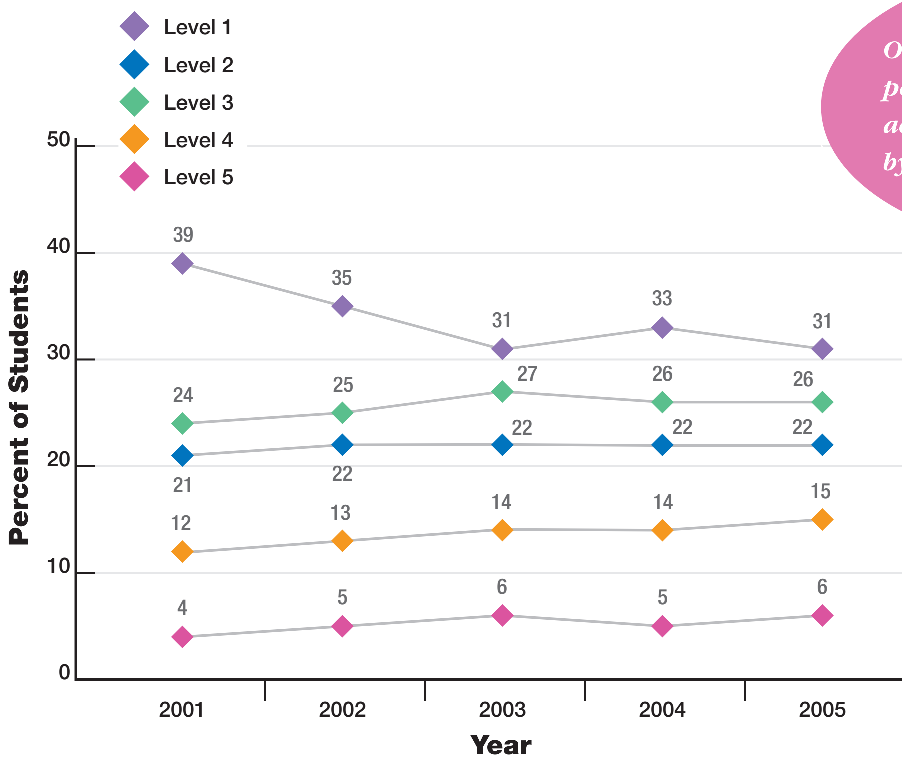
Graph M-14
Mathematics Grade 6
Percent of Students in Levels 3–5



The graph shows a slight increase from 2001 through 2005, moving from 40% to 47%. Results from the last three years (2003 through 2005) were relatively flat. Most recent results show that 47% of students achieved in Level 3 or higher. This was lower relative to Grades 3, 4, and 5, which had corresponding results in 2005 of 68%, 64%, and 57%, respectively.

The following graph provides a closer look at Grade 6 trends for each Achievement Level.

Graph M-15
Mathematics Grade 6
Percent of Students by Achievement Level

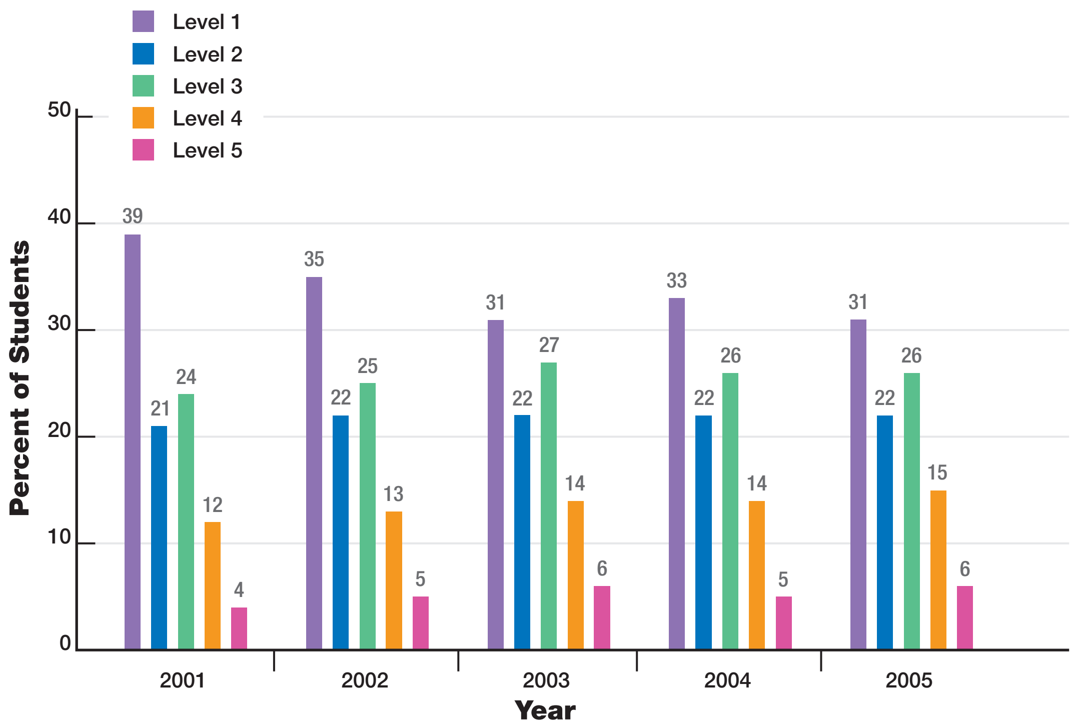


Over the five-year period, the percent of students who achieved in Level 1 decreased by 8 percentage points.

Over the five-year period, the percent of students who achieved in Level 1 decreased by 8 percentage points. This decrease took place between 2001 and 2003; the percents from 2003 through 2005 were fairly flat. The percents of students in all other Achievement Levels (2–5) were stable or increased slightly from 2001 through 2005.

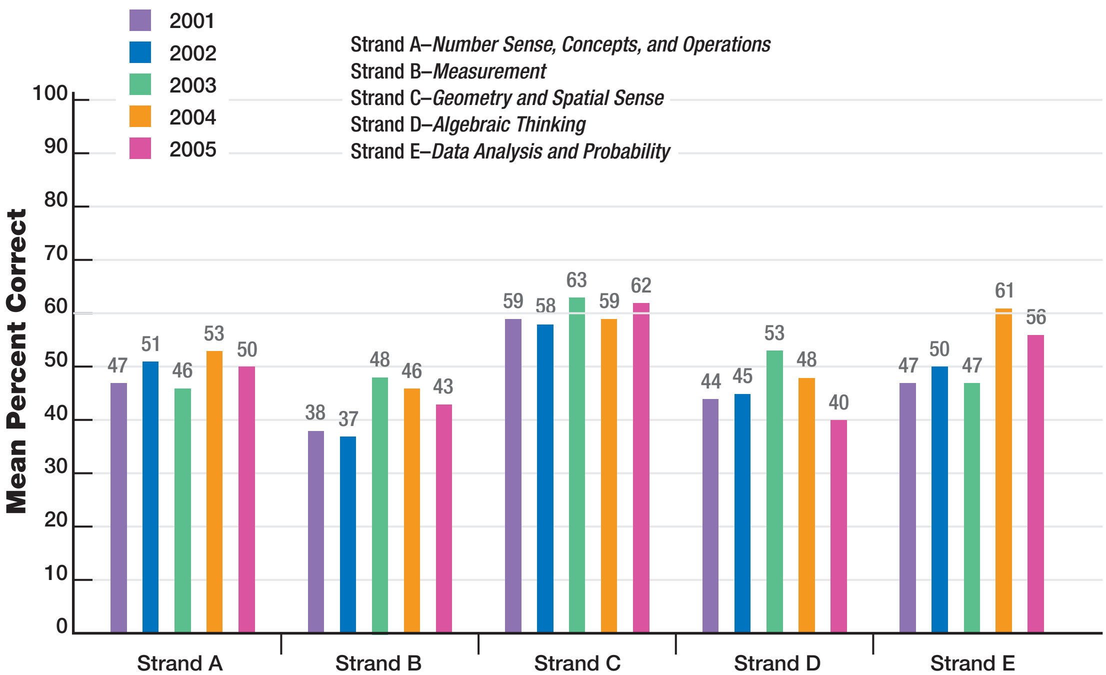
The following bar graph further illustrates a shift that primarily took place in Level 1, with stable results in Level 2, and small percentage-point increases (1% or 2%) in Levels 3, 4, and 5.

Graph M-16
Mathematics Grade 6
Percent of Students by Achievement Level



The following graph illustrates the performance of Grade 6 students across mathematics strands.

Graph M-17
Mathematics Grade 6
Mean Percent Correct by Strand

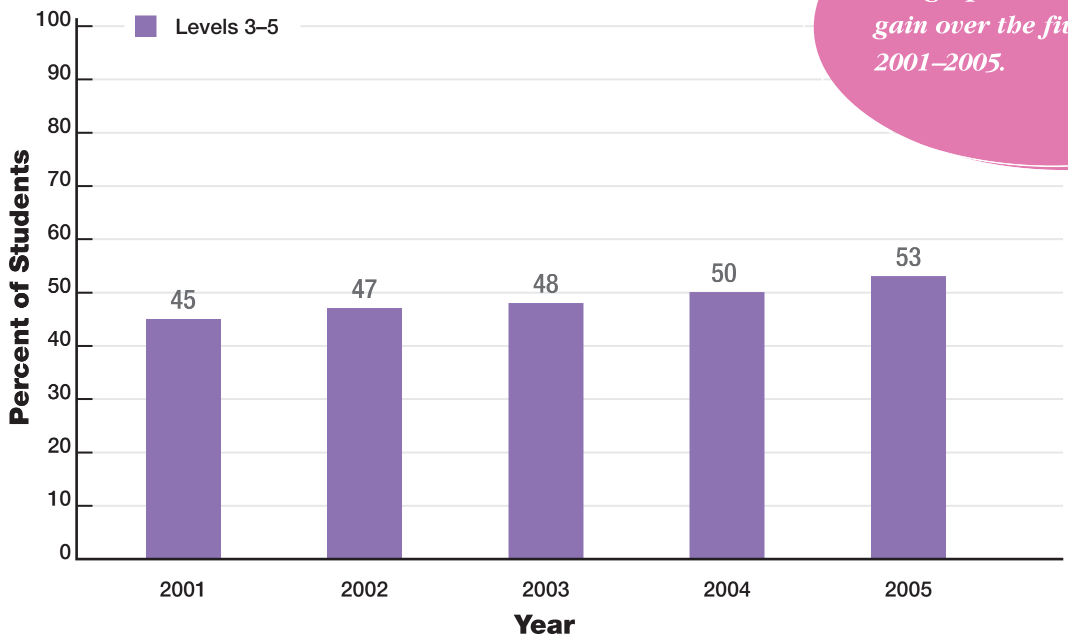


Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

Analysis of FCAT Mathematics Grade 7 Results 2001–2005

The results for the percent of Grade 7 students who achieved in Level 3 or higher are provided in the following graph.

Graph M-18
Mathematics Grade 7
Percent of Students in Levels 3–5

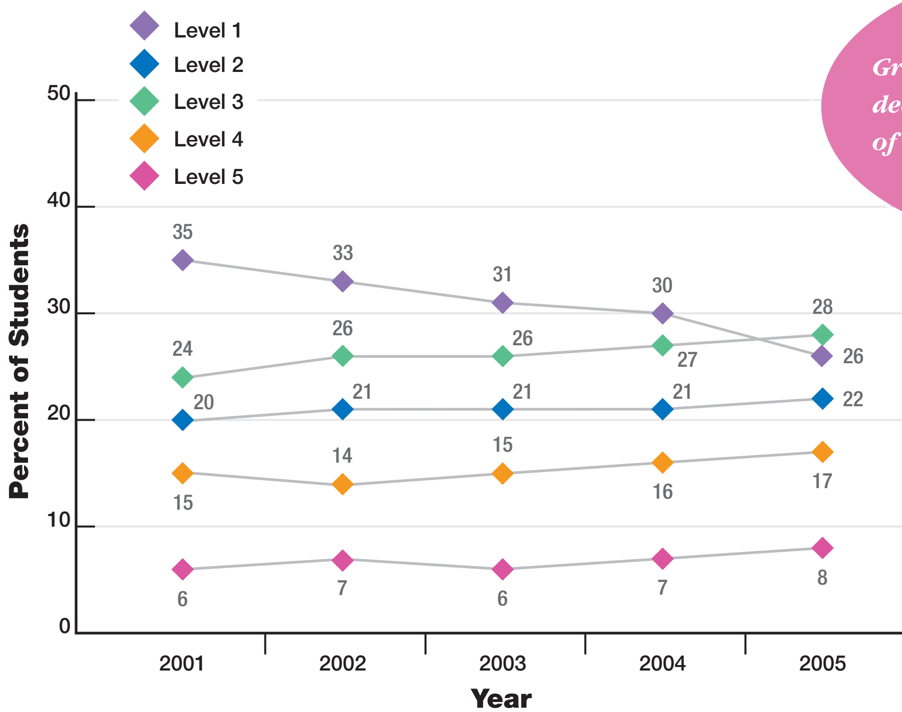


The graph shows a steady gain over the five-year period, 2001–2005.

The graph shows a steady gain over the five-year period. The percent of students who achieved in Levels 3 or higher increased from 45% in 2001 to 53% in 2005, which is an increase of 8 percentage points.

Further details of trends by Achievement Level are provided in the following graph.

Graph M-19
Mathematics Grade 7
Percent of Students by Achievement Level



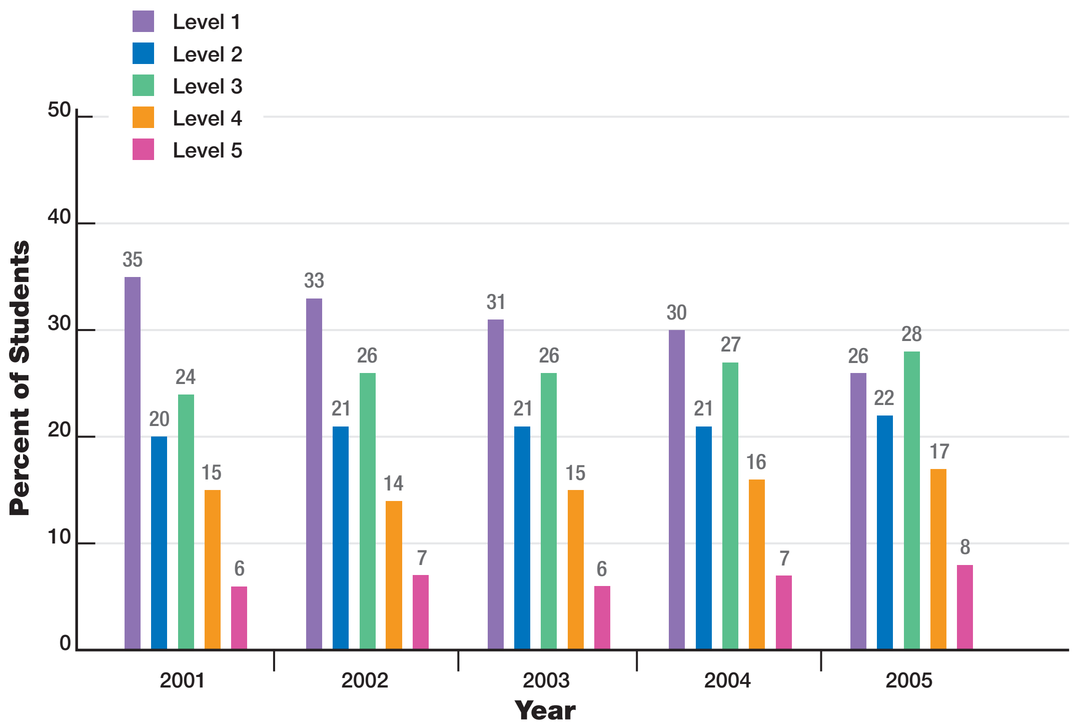
Grade 7 also showed a decreasing trend in the percent of students in Level 1.

Grade 7

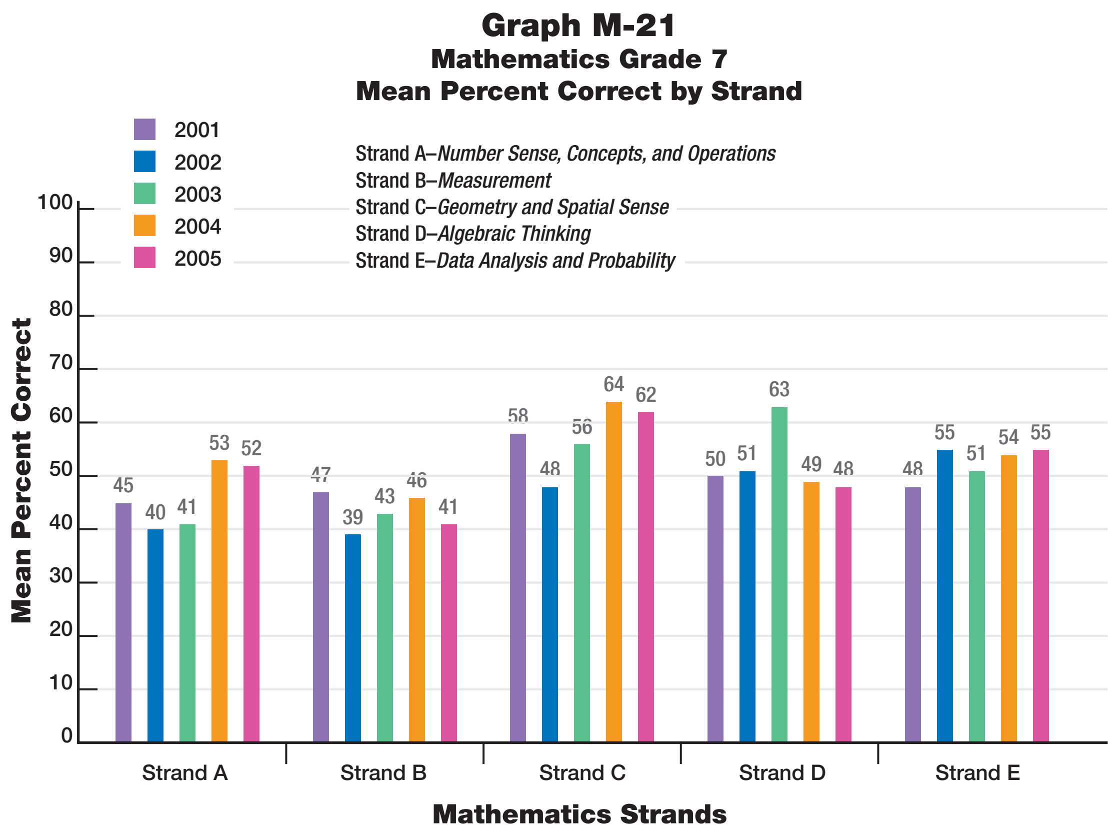
Grade 7 also showed a decreasing trend in the percent of students in Level 1. From 2001 through 2005, there was a decrease of 9 percentage points. The trend for Level 2 was relatively flat with a slight increase of 2 percentage points from 2001 through 2005. It appears that the shift in the distribution of scores from Level 1 was largely to Level 3, where there was an increase of 4 percentage points over the five-year period. The trends in Levels 4 and 5 were relatively flat with increases of 2 percentage points at each level.

The trends identified in the previous graph are further illustrated in the following graph. Like Grade 6 trends, Grade 7 results showed a distributional shift from Level 1 to Levels 3, 4, and 5.

Graph M-20
Mathematics Grade 7
Percent of Students by Achievement Level



The following graph provides Grade 7 student results across strands.

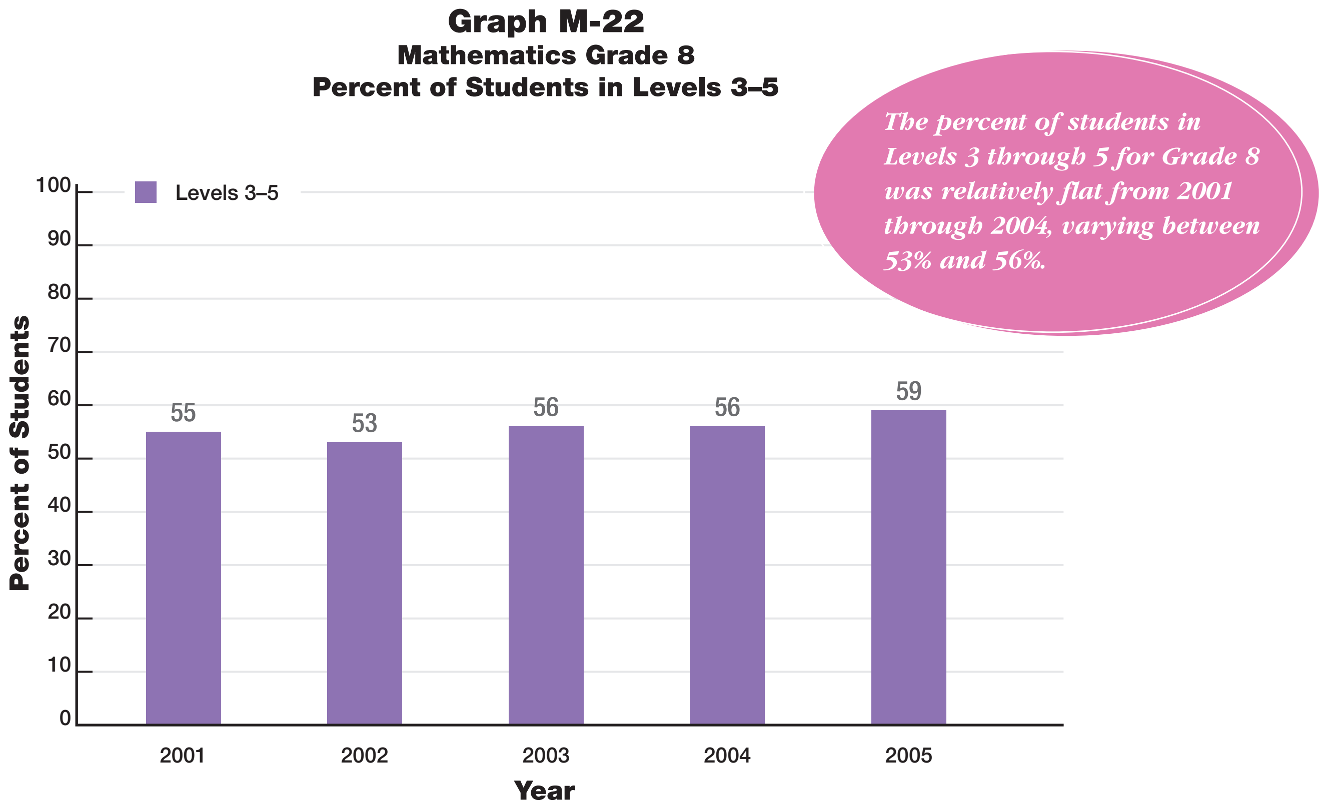


Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

Grade 7

Analysis of FCAT Mathematics Grade 8 Results 2001–2005

The results for the percents of Grade 8 students who achieved in Level 3 or higher are provided in the following graph.

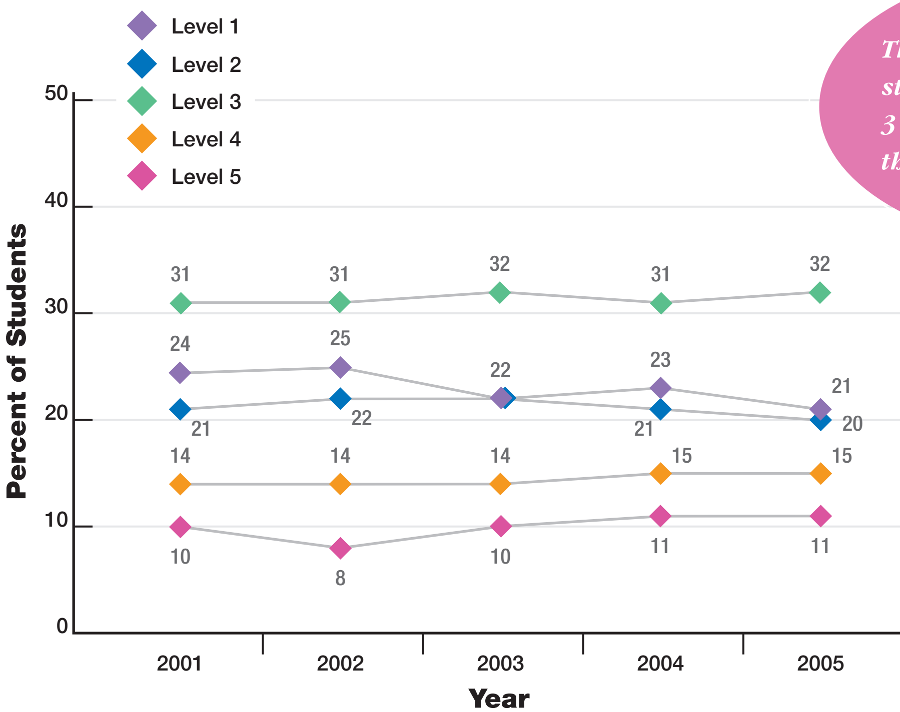


The percent of students in Levels 3 through 5 for Grade 8 was relatively flat from 2001 through 2004, varying between 53% and 56%. There was an overall increase of 4 percentage points from 2001 to 2005.

Grade 8

The following graph illustrates the trends at each Achievement Level over the five-year period.

Graph M-23
Mathematics Grade 8
Percent of Students by Achievement Level



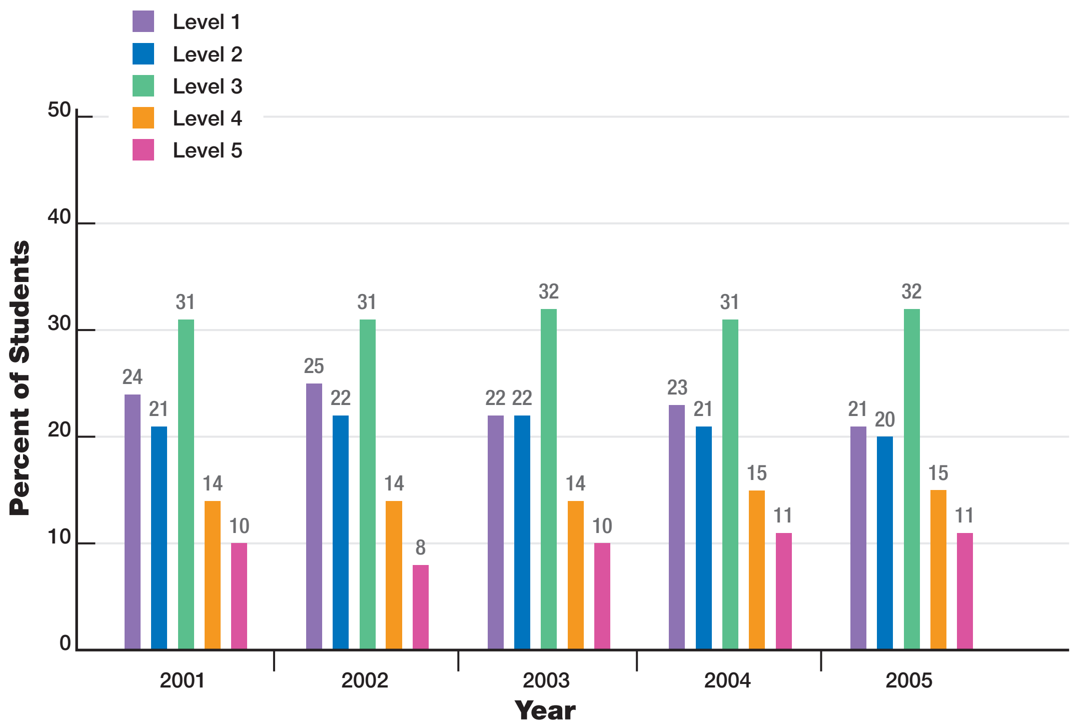
The percent of Grade 8 students in Level 1 decreased by 3 percentage points from 2001 through 2005.

The percent of Grade 8 students in Level 1 decreased by 3 percentage points from 2001 through 2005. Results in Level 2 were relatively flat over the five-year period, decreasing by 1 percentage point from 2001 through 2005. Levels 3, 4, and 5 were relatively stable with 1-percentage-point increases within each level.

Grade 8

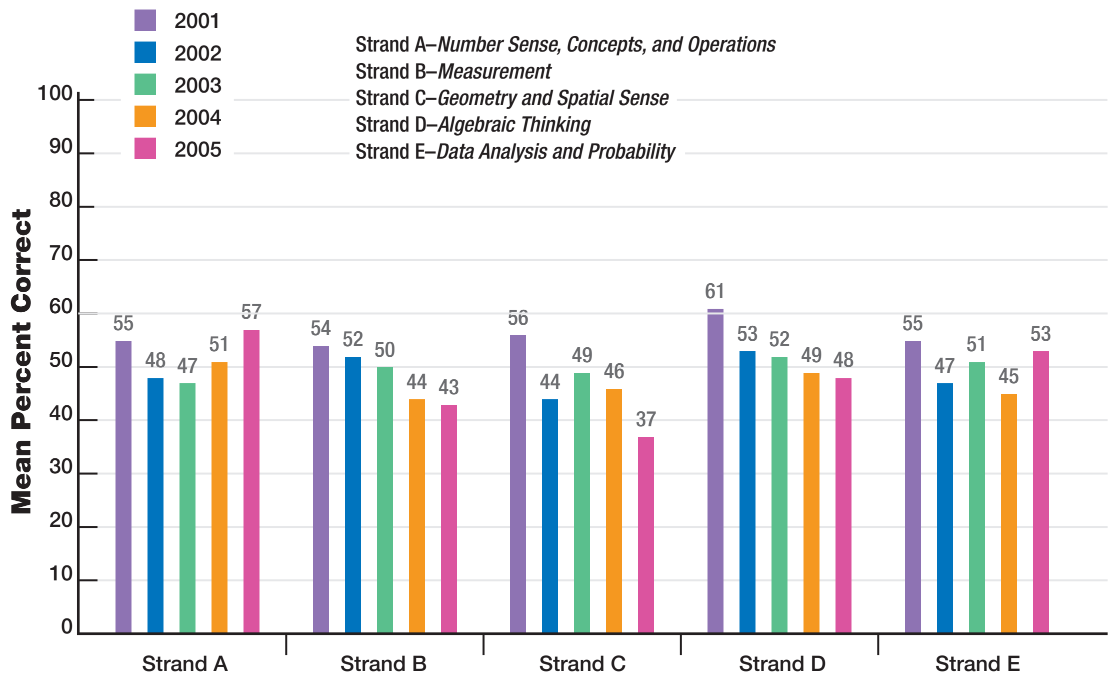
The following bar graph shows the relatively stable distribution of scores from year to year. Again, the exception was a 3-percentage-point decrease in the percent of students in Level 1.

Graph M-24
Mathematics Grade 8
Percent of Students by Achievement Level



The following graph provides results for Grade 8 students across strands.

Graph M-25
Mathematics Grade 8
Mean Percent Correct by Strand

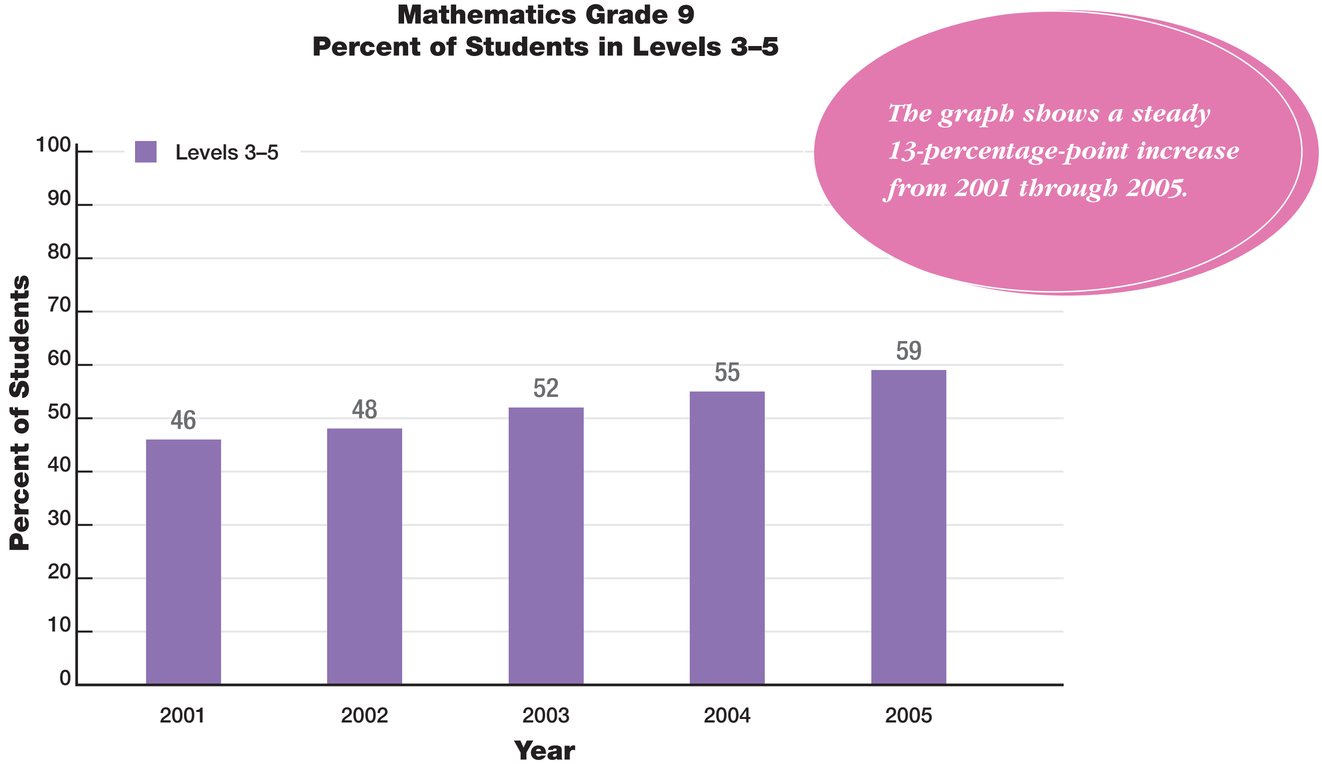


Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

Analysis of FCAT Mathematics Grade 9 Results 2001–2005

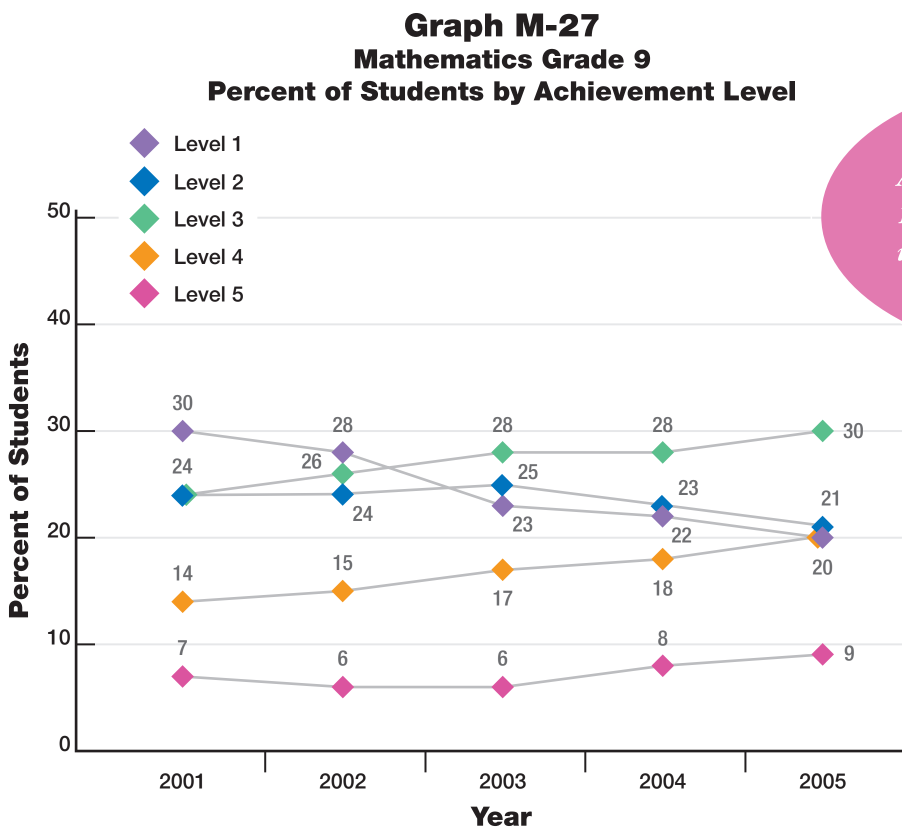
The results for the percent of Grade 9 students who achieved in Level 3 or higher are provided in the following graph.

Graph M-26
Mathematics Grade 9
Percent of Students in Levels 3–5



The graph shows a steady 13-percentage-point increase from 2001 through 2005. Most recent results show that 59% of students achieved in Level 3 or higher.

The following graph illustrates the trend lines for the percents of Grade 9 students at each Achievement Level.

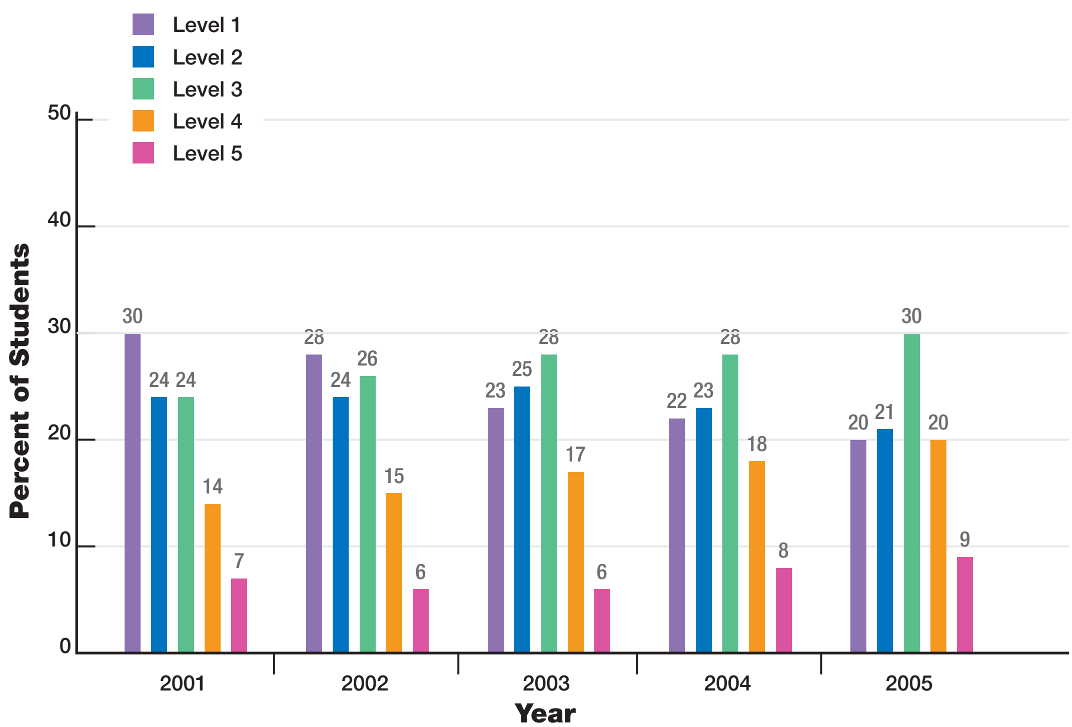


At Grade 9, there was a 10-percentage-point decrease in students in Level 1.

At Grade 9, there was a 10-percentage-point decrease in students in Level 1. The distributions of scores shifted primarily to Levels 3 and 4, with 6-percentage-point increases in each level.

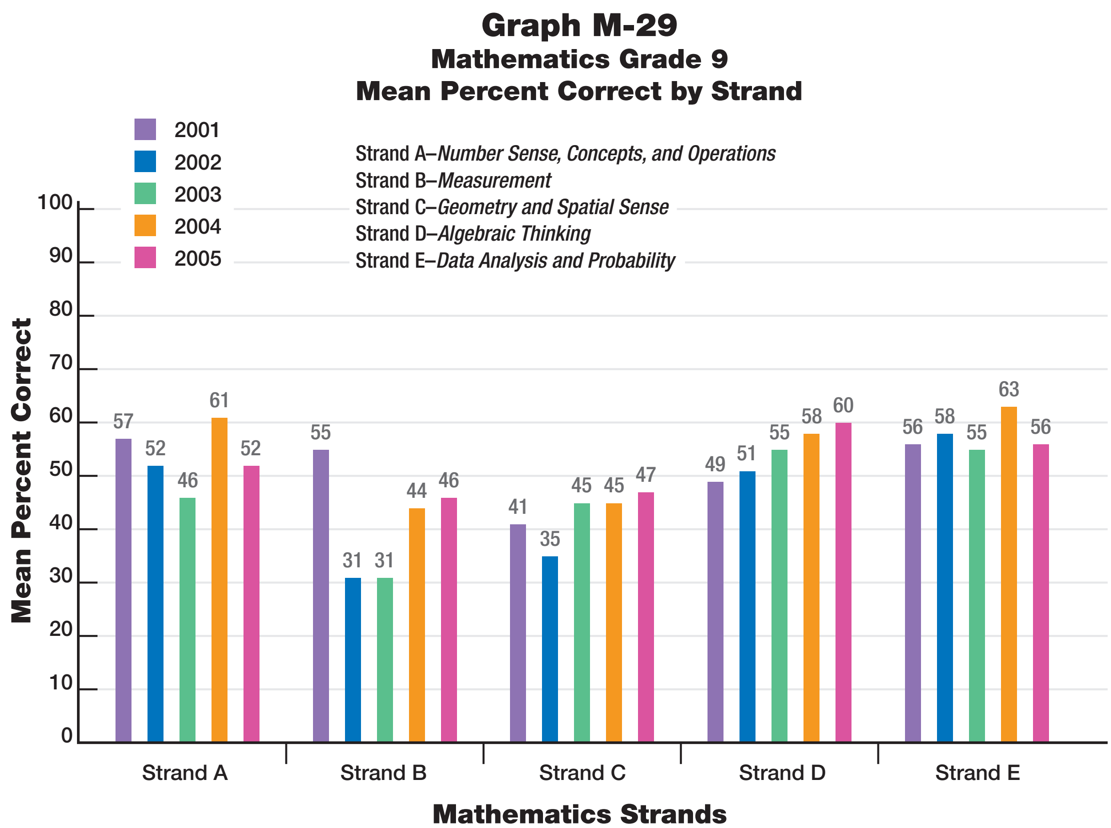
The following graph illustrates the distributional shift from year to year at Grade 9.

Graph M-28
Mathematics Grade 9
Percent of Students by Achievement Level



Grade 9

The following graph illustrates the results for Grade 9 students across strands.

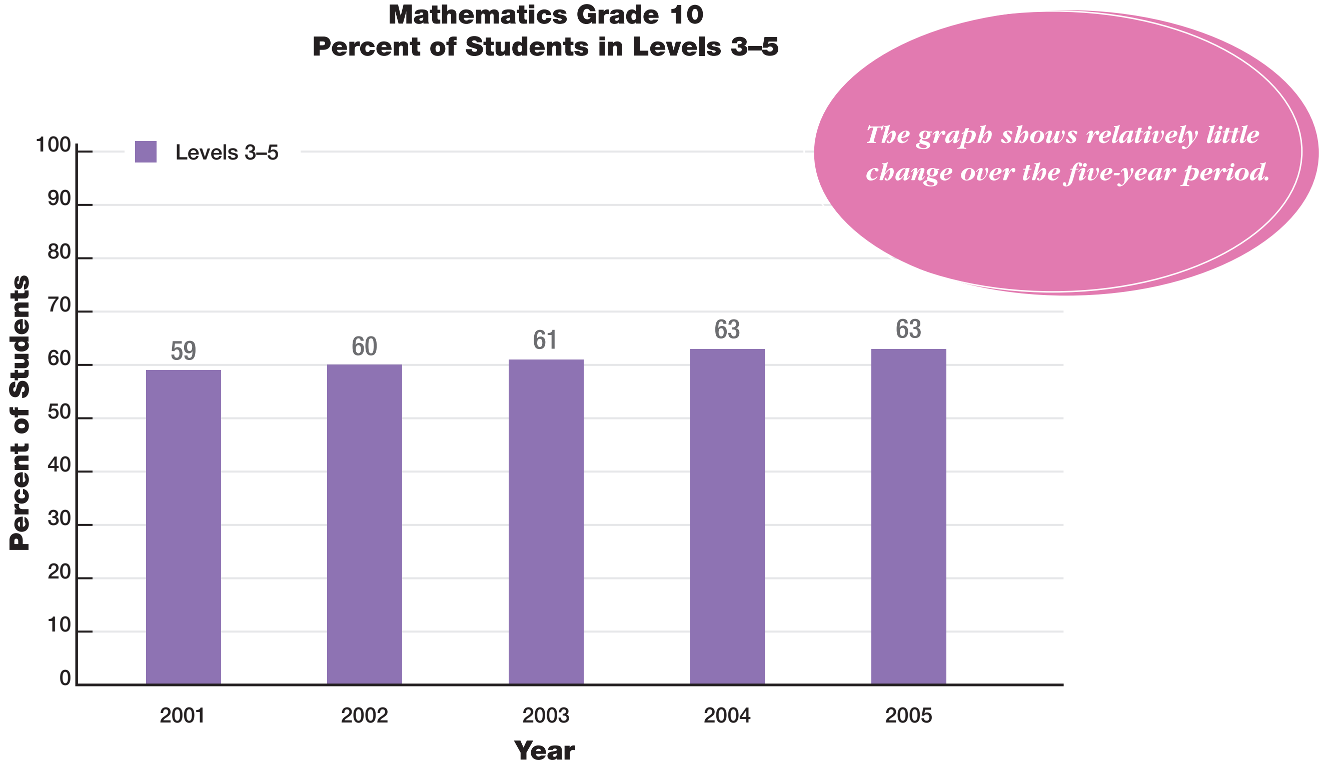


Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

Analysis of FCAT Mathematics Grade 10 Results 2001–2005

The results for the percents of Grade 10 students who achieved in Level 3 or higher are provided in the following graph.

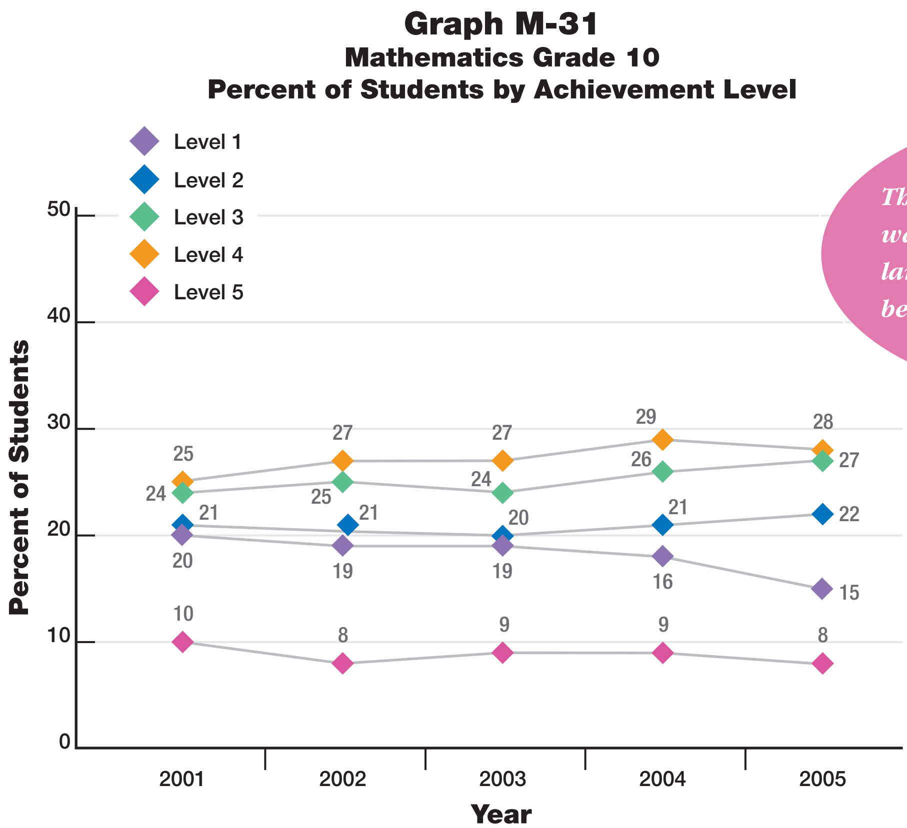
Graph M-30
Mathematics Grade 10
Percent of Students in Levels 3–5



The graph shows relatively little change over the five-year period. The increase from 2001 through 2004 was 4%. Most recent results show that 63% of students achieved in Level 3 or higher.

Grade 10

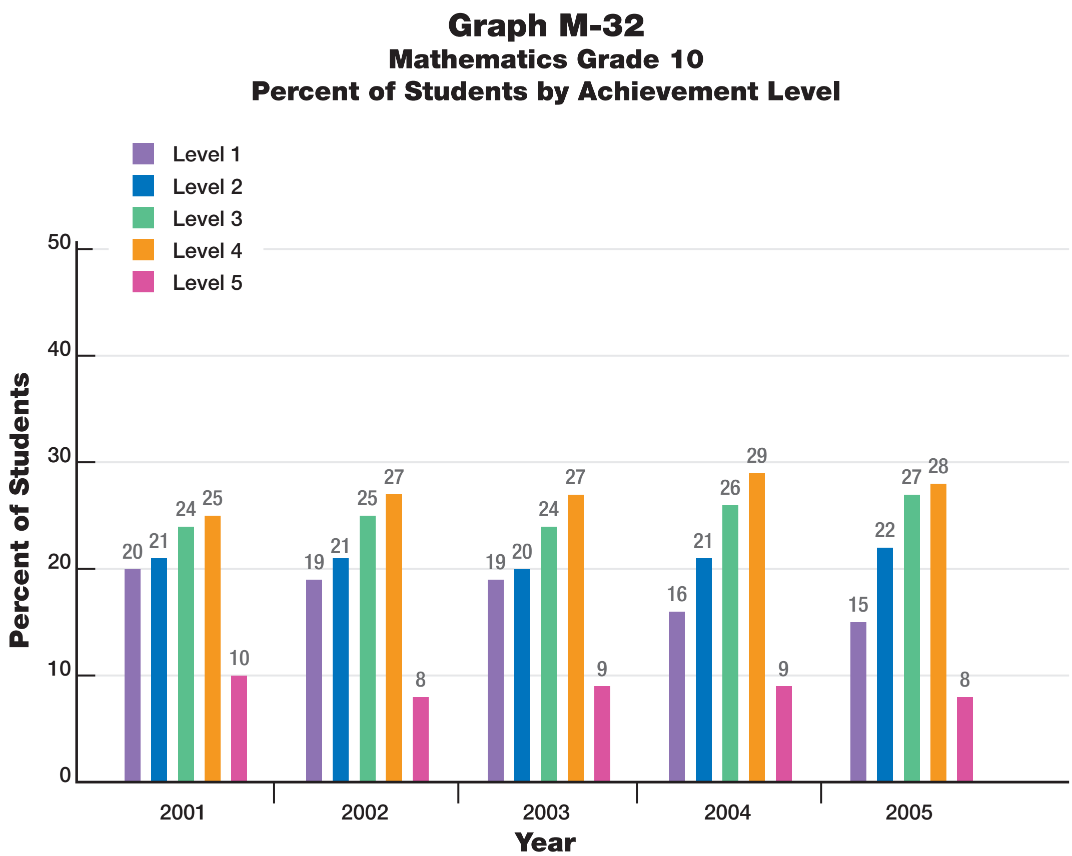
Trend results for each Achievement Level are provided in the following graph.



The decrease in Level 1 scores was 5 percentage points, and the largest decrease occurred between 2003 and 2005.

The decrease in Level 1 scores was 5 percentage points, and the largest decrease occurred between 2003 and 2005. The trend for Level 2 was relatively flat, increasing by 1 percentage point from 2001 through 2005. There was an increase of 3 percentage points in Level 3. The trends for Levels 4 and 5 were relatively flat, with a 3-percentage-point increase in Level 4 and a 2-percentage-point decrease in Level 5.

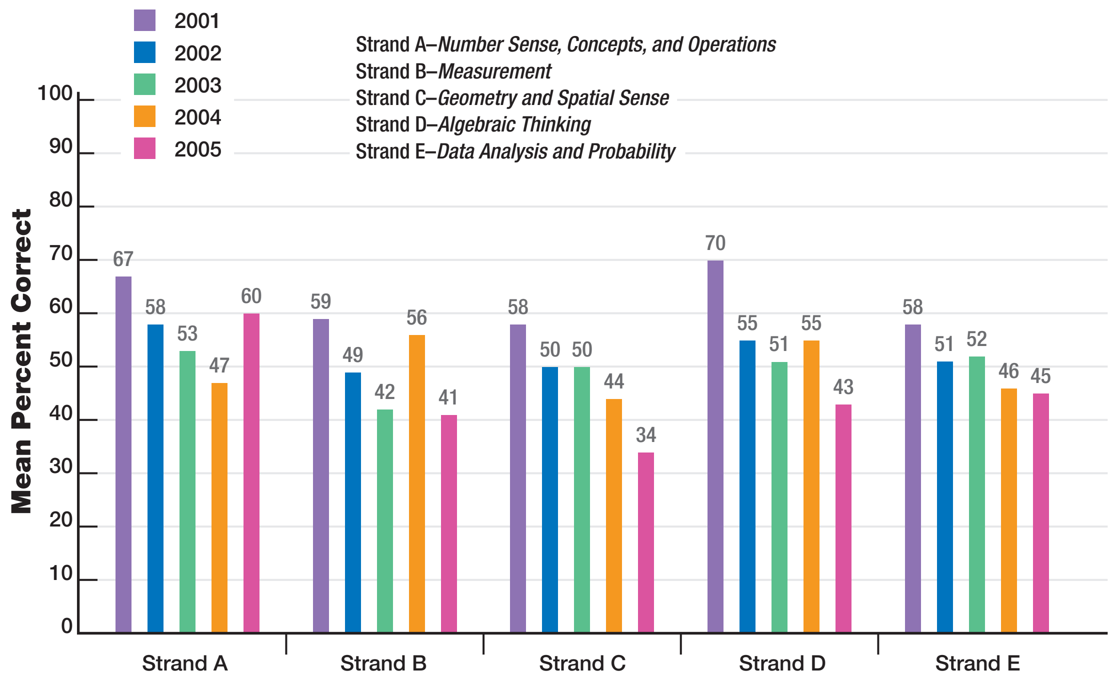
The following graph shows the decrease in the percent of students in Level 1 and the increase in the percent of students in Levels 3 and 4.



Grade 10

Results for Grade 10 students across strands are provided in the following graph.

Graph M-33
Mathematics Grade 10
Mean Percent Correct by Strand



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

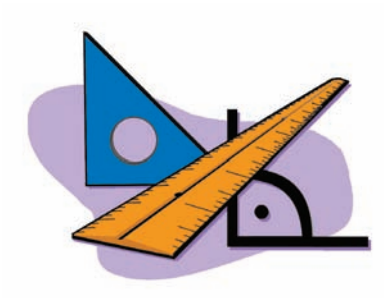
Grade-Level Summary of FCAT Mathematics Performance 2001–2005

The overall trend results provided in the previous sections are summarized in this section by the following grade ranges: Grades 3–5 (elementary), Grades 6–8 (middle school), and Grades 9–10 (high school). This summary is intended to provide the reader with a perspective of student performance and, indirectly, program performance by grade level.

The trend results were mostly favorable for the elementary grades. The increases in the percents of students scoring in Level 3 or higher in Grades 3, 4, and 5 were 16, 19, and 10 percentage points, respectively.

In the middle school grades, increases in the percents of students scoring in Level 3 or higher were 7%, 8%, and 4% for Grades 6, 7, and 8, respectively. These increases were less than those at the elementary grade levels.

In the high school grades, greater gains in achievement occurred with students in Grade 9. The increases in the percents of students scoring in Level 3 or higher were 13 percentage points for Grade 9, and 4 percentage points for Grade 10.



Lessons Learned LONGITUDINAL RESULTS

FCAT Mathematics Statewide Achievement Longitudinal Results

Longitudinal analyses track the progress of a cohort of students from grade to grade. For example, this *Lessons Learned* report shows Grade 3 results in 2001, Grade 4 results in 2002, Grade 5 results in 2003. This group of students is referred to as the Grade 3, 2001 cohort.

Perfectly matched cohorts require the researcher to account for students who move in and out of state, thereby limiting the analysis to those students who were in the cohort during the time period studied. While this analysis did not use such perfectly matched cohorts, average achievement results at the state level will be unaffected, largely due to the substantial volume of data. In schools or districts with fewer students, the accuracy of longitudinal results may become more questionable given that changes in smaller student populations affect average results more.

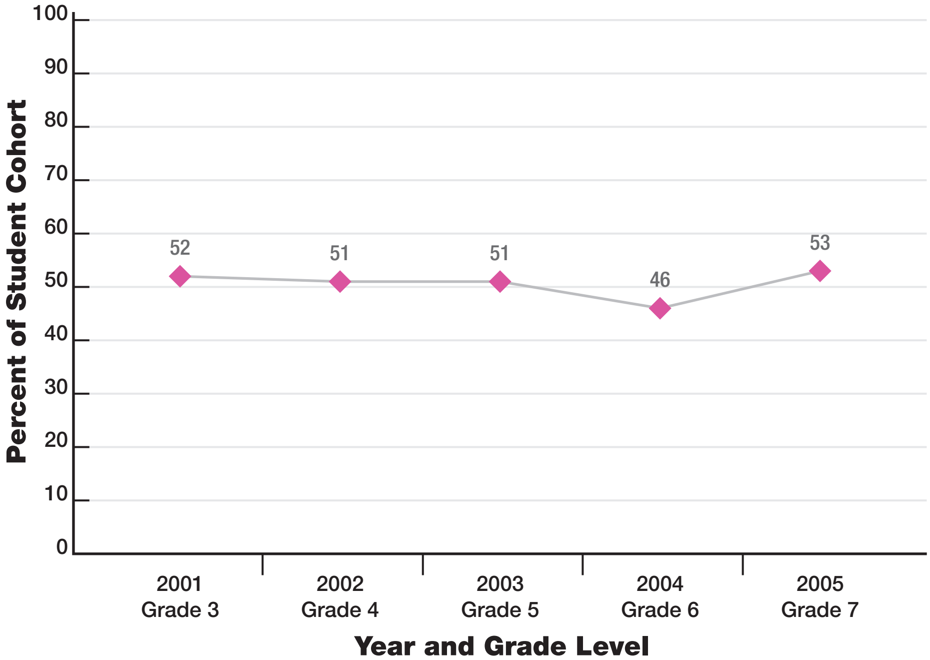
There are two premises that should be considered when interpreting the results of this section:

- Two different Standards Setting Committees established the Achievement Levels used at each grade. The score required for achieving Level 3 in Grade 10, for example, appears to be more challenging for students to attain than that required for Level 3 in Grade 8.
- Cohort information for Grades 9 and 10 will not be provided because these cohorts are too limited to report (i.e., the Grade 9 cohort information is limited to Grade 9 achievement and then Grade 10 achievement in the following year). It is not advisable to suggest a trend with fewer than three years of results.

Analysis of FCAT Mathematics Grade 3 Results 2001–2005

The following graph provides the longitudinal trend results for the Grade 3, 2001 cohort of students. This is not a perfectly matched cohort but will be referred to as *the cohort* for ease of reference.

Graph M-34
Mathematics Grade 3, 2001 Cohort
Percent of Students in Levels 3–5



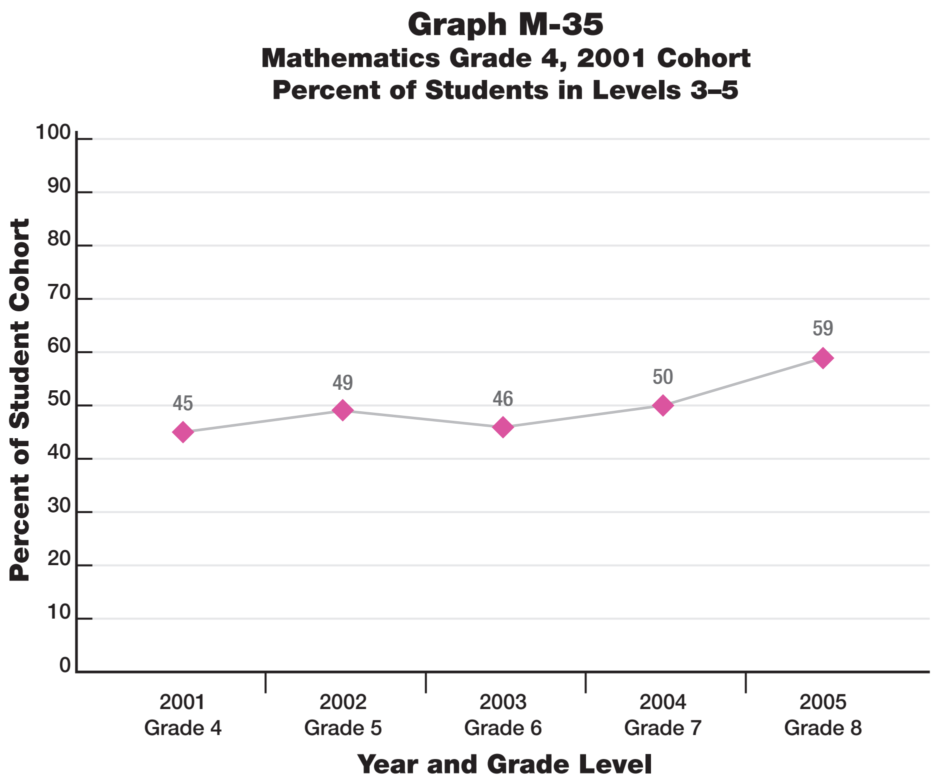
The graph can be interpreted as follows:

- 52% of the cohort scored in Level 3 or higher on the Grade 3 assessment in 2001.
- 51% of the cohort scored in Level 3 or higher on the Grade 4 assessment in 2002.
- 51% of the cohort scored in Level 3 or higher on the Grade 5 assessment in 2003.
- 46% of the cohort scored in Level 3 or higher on the Grade 6 assessment in 2004.
- 53% of the cohort scored in Level 3 or higher on the Grade 7 assessment in 2005.

The Grade 3, 2001 cohort showed little increase in terms of percent of students scoring in Level 3 or above.

Analysis of FCAT Mathematics Grade 4 Results 2001–2005

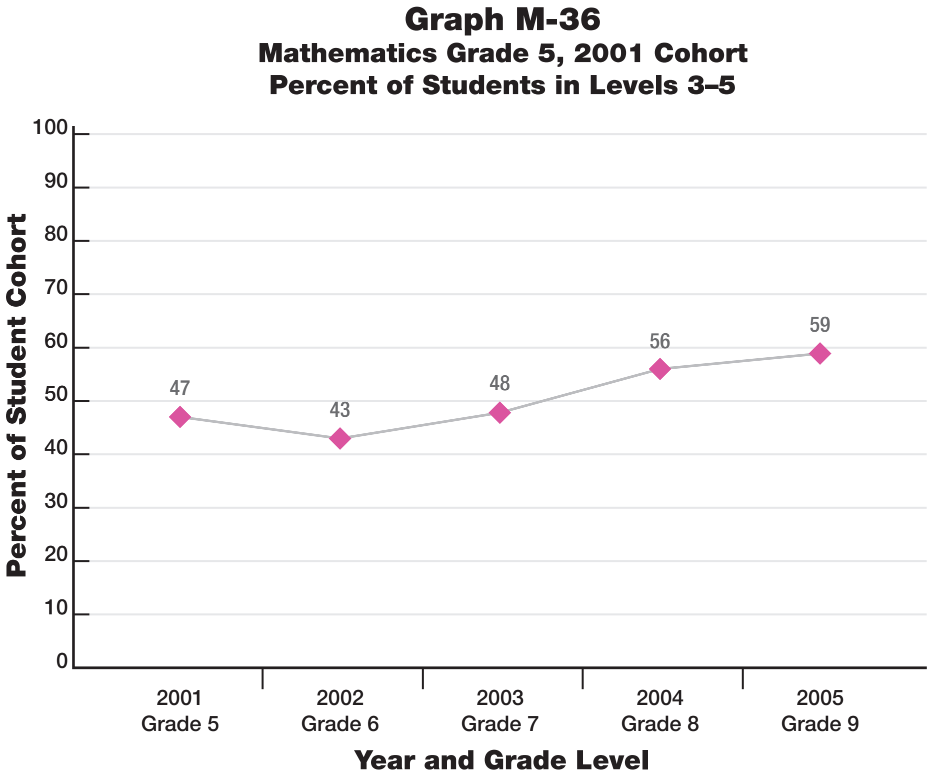
The following graph provides the longitudinal trend results for the Grade 4, 2001 cohort.



From 2003 through 2005, there was a steady increase in students reaching Level 3 or higher. The cohort started at 45% in 2001 and increased by 14 percentage points to 59% in 2005.

Analysis of FCAT Mathematics Grade 5 Results 2001–2005

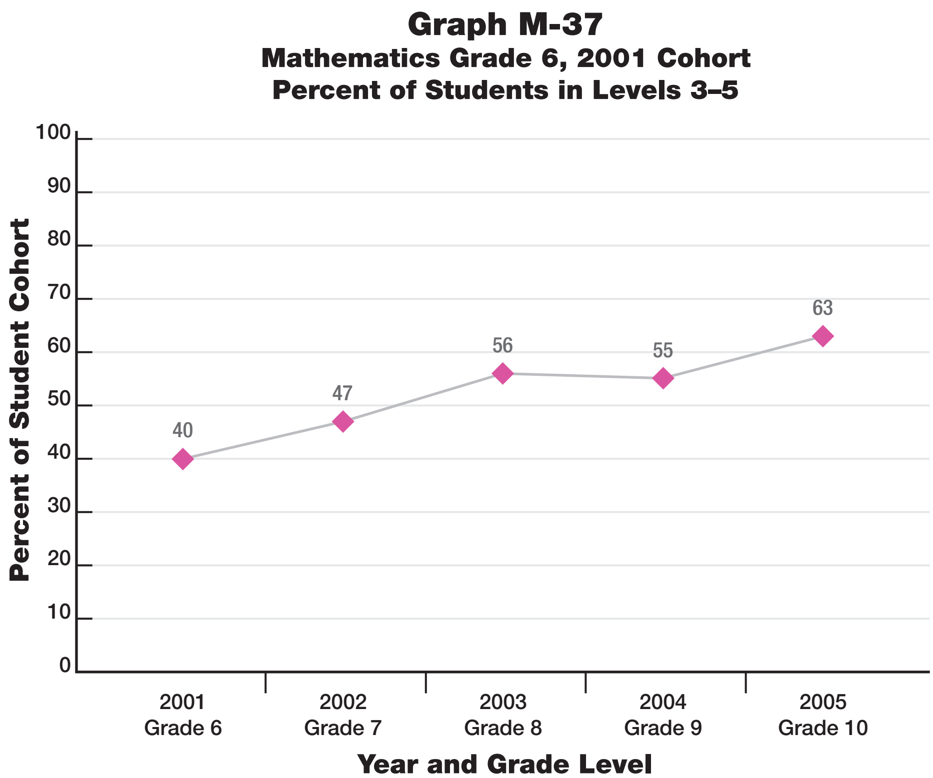
The following graph provides the longitudinal trend results for the Grade 5, 2001 cohort.



Like the Grade 4 cohort, the Grade 5, 2001 cohort showed an increase in the percent of students scoring in Level 3 or higher. From 2002 through 2005, there was a steady increase of 16 percentage points. It is worth noting that for each of the cohorts shown in Graphs M-34, M-35, and M-36, the percent of students in Level 3 and higher decreased as each cohort moved from Grade 5 to Grade 6.

Analysis of FCAT Mathematics Grade 6 Results 2001–2005

The following graph provides the longitudinal trend results for the Grade 6, 2001 cohort.

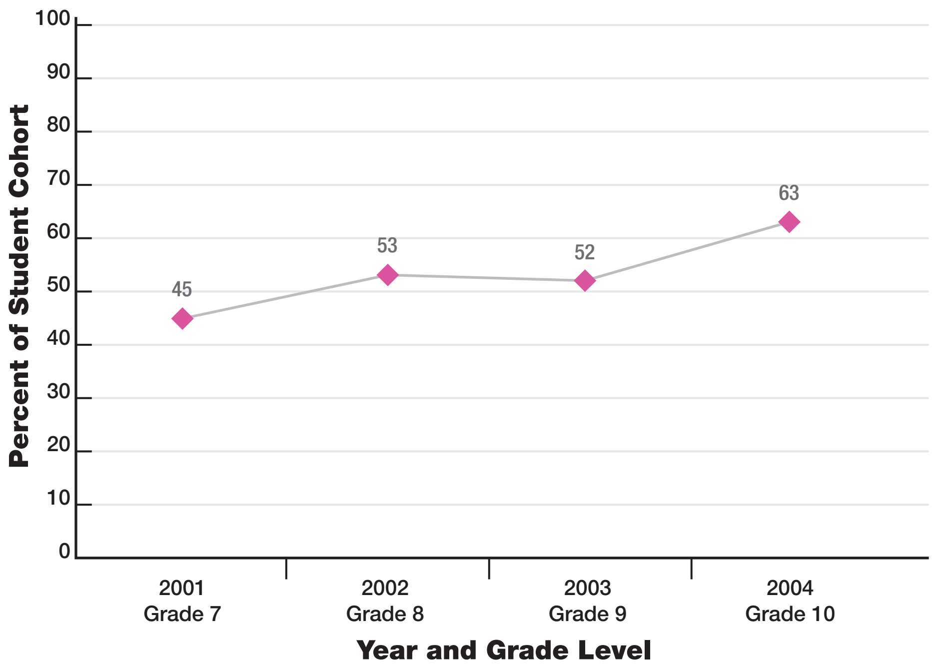


From 2001 through 2005, the Grade 6, 2001 cohort showed a positive trend; the percent of students in Level 3 or higher increased by 23 percentage points. It should be noted that this was the largest increase of all cohorts analyzed.

Analysis of FCAT Mathematics Grade 7 Results 2001–2005

The longitudinal trend results for the Grade 7, 2001 cohort are provided in the following graph. Because the last result available for the Grade 7, 2001 cohort is Grade 10 information from 2004, the graph only covers four years.

Graph M-38
Mathematics Grade 7, 2001 Cohort
Percent of Students in Levels 3–5

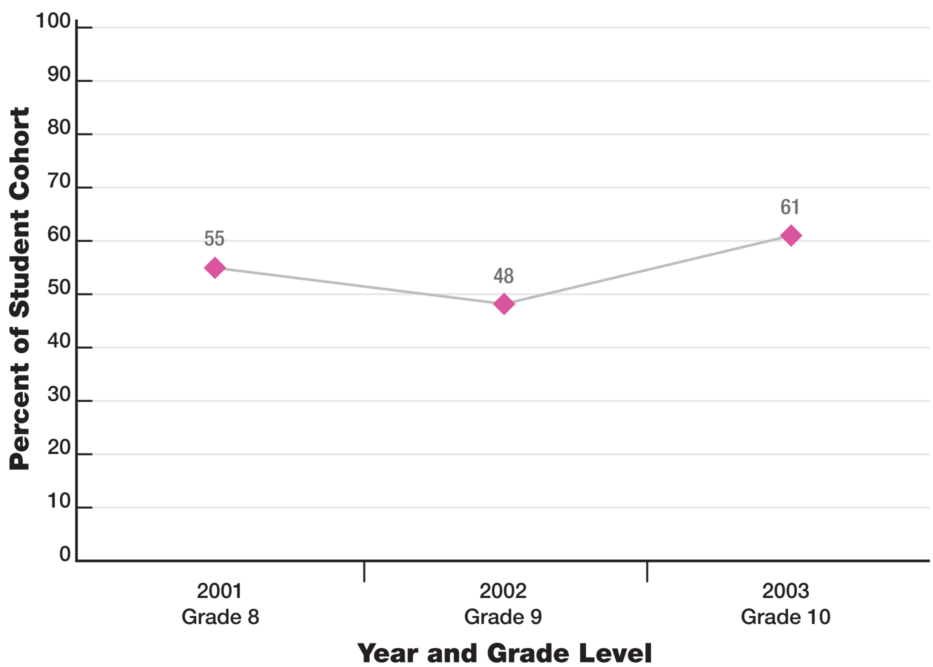


In the four-year period, the Grade 7, 2001 cohort demonstrated an 18-percentage-point increase in Achievement Levels 3 and higher, from 45% to 63%. The largest increases took place in the 2002 and 2004 FCAT administrations.

Analysis of FCAT Mathematics Grade 8 Results 2001–2005

The final longitudinal graph provides trend results for the Grade 8, 2001 cohort. The range of this graph is 2001 through 2003, which covers Grades 8, 9, and 10 for the 2001 cohort.

Graph M-39
Mathematics Grade 8, 2001 Cohort
Percent of Students in Levels 3–5



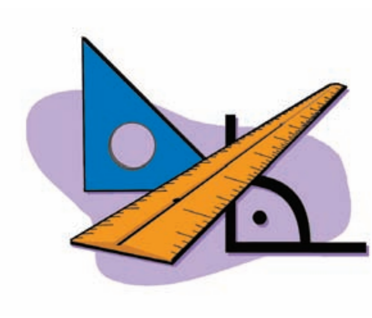
While there is no clear trend for this cohort, it is important to note that there was a 13-percentage-point increase in the percent of students in Achievement Levels 3 and higher, from 2002 through 2003. For each of the cohorts shown in Graphs M-37, M-38, and M-39, the percent of students in Levels 3 and higher decreased as each cohort moved from Grade 8 to Grade 9. The only cohort not experiencing such a drop was the Grade 5, 2001 cohort (Graph M-36 on page 58).

Longitudinal Analysis Summary

The results of the longitudinal analysis are summarized in the following table.

Table 9: Mathematics Longitudinal Cohort Percentage Point Change						
2001 Grade Cohort	Percent of Students in Levels 3–5					Overall Percentage Point Change
	2001	2002	2003	2004	2005	
3	52%	51%	51%	46%	53%	1%
4	45%	49%	46%	50%	59%	14%
5	47%	43%	48%	56%	59%	12%
6	40%	47%	56%	55%	63%	23%
7	45%	53%	52%	63%		18%
8	55%	48%	61%			6%

The table shows that the cohort demonstrating the biggest gain (23 percentage points) was the Grade 6, 2001 cohort. In 2001, 40% of students in the Grade 6 cohort performed in Level 3 or higher. The percent of students in this group performing in Level 3 or higher increased to 63% in 2005. The smallest growth (1 percentage point), in terms of the percent of students achieving in Level 3 or higher, was realized by the Grade 3, 2001 cohort. The reader should keep in mind that comparison to the other grades is difficult when there are fewer than five years to show improvement.



Lessons Learned
RESULTS BY STRAND & INSTRUCTIONAL IMPLICATIONS

FCAT Mathematics Statewide Achievement Results by Strand with Instructional Implications

This section of *Lessons Learned* includes an analysis of the performance by strand and grade level (Grades 3–5, 6–8, and 9–10). The examination of each strand across levels provides important instructional implications to educators. Teachers at the elementary and middle school levels can benefit from learning about the expectations for student performance through high school. Finally, a view of student performance through the lens of achievement on strands provides valuable information on similarities and differences across grades.

The task force examined student performance on the five strands and on the individual benchmarks that make up those strands to determine particular areas of student strength or weakness, the types of errors that limited student performance, and appropriate instructional strategies for classroom teachers. They supplemented these analyses with an examination of student performance on individual test questions at the SSS standard level. Sample test questions are interspersed throughout the book to highlight observations for given strands and to demonstrate the variety of content assessed, though not all grades have sample items for each strand. The percent of students who selected each option or received a given rubric score is provided for each of the sample questions. In some cases, these percents will not add up to 100%. This is a result of rounding each percent to a whole number.

The task force provided implications for instruction by grade level and standard within a given strand.

To gain a better understanding of student performance, the task force analyzed both the correct answers and the incorrect answers that proved most attractive to students who did not respond correctly. For the purposes of *Lessons Learned*, students were deemed to have been successful when approximately 50% or more of the students responded correctly to an item. For additional information about the characteristics of FCAT items, including difficulty level, see Section 4.4 of the *FCAT Handbook* at http://fcat.fldoe.org/handbk/fcathandbook.asp.

For each strand, mean percent correct student performance data are presented. The task force provided implications for instruction by grade level and standard within a given strand. Sample test questions further clarify these implications.

Each strand analysis section is organized by

- the strand title;
- a graph depicting student performance data for that strand at each grade level;
- a chart showing strand benchmarks by grade; and
- a summary of performance, with a sample question, and implications for instruction.

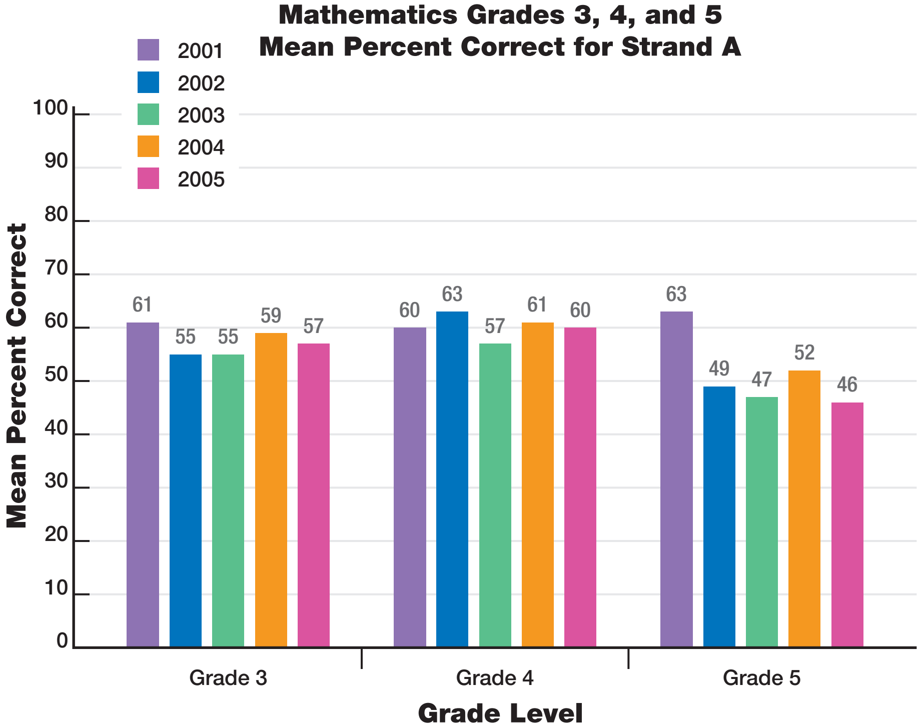
Strand A—*Number Sense, Concepts, and Operations*

Grades 3–5

Strand A Results for Grades 3–5

The Grades 3–5 results on Strand A (*Number Sense, Concepts, and Operations*) are displayed in the graph below.

Graph M-40
Mathematics Grades 3, 4, and 5
Mean Percent Correct for Strand A



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the Strand A (*Number Sense, Concepts, and Operations*) standards and benchmarks for Grades 3–5. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Standard A1: The student understands the different ways numbers are represented and used in the real world.
Benchmark MA.A.1.2.1: The student names whole numbers combining three-digit numeration (hundreds, tens, ones) and the use of number periods, such as ones, thousands, and millions and associates verbal names, written word names, and standard numerals with whole numbers, commonly used fractions, decimals, and percents. (Assessed with A.1.2.4)
Benchmark MA.A.1.2.2: The student understands the relative size of whole numbers, commonly used fractions, decimals, and percents.
Benchmark MA.A.1.2.3: The student understands concrete and symbolic representations of whole numbers, fractions, decimals, and percents in real-world situations. (Assessed with A.1.2.4)
Benchmark MA.A.1.2.4: The student understands that numbers can be represented in a variety of equivalent forms using whole numbers, decimals, fractions, and percents. (Also assesses A.1.2.1 and A.1.2.3)

Grade 3: Observations for Standard A1–*Representing Numbers*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- compare the relative size of whole numbers; and
- translate the written name of whole numbers to standard form.

Students who are **unsuccessful** have the greatest difficulty with
- finding an equivalent fraction from a graphic representation of a set of objects (e.g., if 2 of 4 marbles are striped, $\frac{1}{2}$ represents the fraction of the set of marbles that are striped).

Grade 4: Observations for Standard A1–*Representing Numbers*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- compare the relative size of whole numbers; and
- compare the relative size of whole numbers and decimals from a table.

Students who are **unsuccessful** have the greatest difficulty with
- comparing fractions and decimals to determine their relative sizes.

Grade 5: Observations for Standard A1–*Representing Numbers*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- convert common fractions such as $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{10}$ to percents or decimals (see sample item below).

Students who are **unsuccessful** have the greatest difficulty with

- comparing the relative size of fractions and mixed numbers with unlike denominators;
- comparing the relative size of numbers with decimals (e.g., 2.36 vs. 2.4);
- converting percents to fractions; and
- converting less-commonly seen fractions such as $\frac{5}{16}$ to percents or decimals.

Tom had a set of colored pencils. He saw that $\frac{1}{5}$ of the pencils in the box were a shade of blue. Which decimal is **equivalent** to $\frac{1}{5}$?

A. 0.15
☞ B. 0.20
C. 0.25
D. 0.50

☞ Correct answer

Most recent student results

29% chose option A
52% chose option B
10% chose option C
9% chose option D

Grades 3–5: Implications for Instruction for Standard A1–*Representing Numbers*

Students should be provided with more in-depth experience using fractions. This may include activities with manipulatives, games and puzzles, and visual aids using fraction circles, fraction bars, hundreds charts, and number lines. Number lines in Grades 4 and 5 should include whole numbers, fractions, and decimals.

Standard A2: The student understands number systems.
Benchmark MA.A.2.2.1: The student uses place-value concepts of grouping based upon powers of ten (thousandths, hundredths, tenths, ones, tens, hundreds, thousands) within the decimal number system.
Benchmark MA.A.2.2.2: The student recognizes and compares the decimal number system to the structure of other number systems such as the Roman numeral system or bases other than ten. (Not assessed)

Grades 3–5: Observations for Standard A2–*Number Systems*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify place values; and
- translate numbers from a written form to a standard form.

The items and student results reviewed by the task force did not warrant any specific observations about areas in which students were unsuccessful.

Grades 3–5: Implications for Instruction for Standard A2–*Number Systems*

The *Lessons Learned* task force determined that students generally did well in this benchmark. The task force recommends that students should continue to work with decimals in place value charts.

Standard A3: The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.

Benchmark MA.A.3.2.1: The student understands and explains the effects of addition, subtraction, and multiplication on whole numbers, decimals, and fractions, including mixed numbers, and the effects of division on whole numbers, including the inverse relationship of multiplication and division.

Benchmark MA.A.3.2.2: The student selects the appropriate operation to solve specific problems involving addition, subtraction, and multiplication of whole numbers, decimals, and fractions, and division of whole numbers.

Grade 3: Observations for Standard A3–*Operations*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- translate word problems to expressions;
- choose the correct operations to solve word problems; and
- solve simple (one-step) addition or multiplication word problems, especially when key words are used in the problems.

Students who were **unsuccessful** have the greatest difficulty
- solving subtraction problems involving regrouping.

Grade 4: Observations for Standard A3–*Operations*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- translate word problems to expressions;
- choose the correct operations to solve word problems; and
- solve simple (one-step) addition or multiplication word problems, especially when key words are used in the problems.

Students who are **unsuccessful** have the greatest difficulty with
- solving multiple-step or two-operation word problems;
- solving subtraction problems that involve regrouping;
- understanding the use of multiplication for converting one unit of measure to another within the same system; and
- understanding the meaning of multiplication in the context of a word problem having extraneous information (see sample item on the following page).

 FCAT Mathematics Lessons Learned |

In an orchestra with 100 musicians, about 60 of the musicians play stringed instruments. The others play woodwind, brass, or percussion instruments. Which expression could be used to find out about how many musicians play stringed instruments in 8 orchestras?

A. $100 - 60$
B. 100×8
C. $60 \div 8$
☞ D. 60×8

☞ Correct answer

Most recent student results

31% chose option A
19% chose option B
20% chose option C
31% chose option D

Grade 5: Observations for Standard A3–*Operations*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- translate word problems into expressions;
- choose the correct operations to solve word problems;
- solve simple (one-step) addition or multiplication word problems, especially when key words are used in the problems; and
- add decimal mixed numbers (e.g., 1.4 + 1.5).

Students who are **unsuccessful** have the greatest difficulty with
- applying the distributive and commutative properties;
- applying order of operations; and
- solving multiple-step problems with fractions and decimals.

Grades 3–5: Implications for Instruction for Standard A3–*Operations*

Students performed well overall, but those who struggled had difficulty understanding the concepts of division and subtraction. Given this, and other findings, the task force recommends more practice with word problems that have multiple steps and problems that do not contain key phrases, such as *how much in all, what is the total,* and *how many are left*. In Grades 3 and 4, students need practice regrouping with problems involving subtraction.

Strand A

Standard A4: The student uses estimation in problem solving and computation.

Benchmark MA.A.4.2.1: The student uses and justifies different estimation strategies in a real-world problem situation and determines the reasonableness of results of calculations in a given problem situation. (Also assesses B.3.2.1)

Grade 3: Observations for Standard A4–*Estimation*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific observations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with

- finding an estimate when given a range of numbers (see sample item below).

Mr. Martin can paint between 27 and 35 bird houses in one day.

Which is the **best estimate** of the total number of bird houses he can paint in 3 days?

A. 30
B. 40
C. 70
☞ D. 90

☞ Correct answer

Most recent student results

19% chose option A
11% chose option B
31% chose option C
40% chose option D

Grade 4: Observations for Standard A4–*Estimation*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific observations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with

- finding estimates using strategies other than rounding; and
- finding an estimate when given a range of numbers (see Grade 3 sample item on previous page).

Grade 5: Observations for Standard A4–*Estimation*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific observations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with

- finding estimates using strategies other than rounding;
- finding an estimate when given a range of numbers; and
- determining an estimate as a range of numbers (e.g., the cost of 4 dozen roses at $1.49 per rose would total between $70 and $75).

Grades 3–5: Implications for Instruction for Standard A4–*Estimation*

The task force's recommendation is that students need more practice estimating. Particularly, teachers should teach estimation strategies that go beyond rounding (e.g., front end, clustering, grouping, and benchmarking). Teachers should not interchangeably use estimation and rounding terminology so that students will not identify rounding as the only viable estimation strategy.

Standard A5: The student understands and applies theories related to numbers.

Benchmark MA.A.5.2.1: The student understands and applies basic number theory concepts, including primes, composites, factors, and multiples.

Grade 3: Observations for Standard A5–*Number Theory Concepts*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific observations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with
- understanding and finding factors and multiples of numbers.

Grade 4: Observations for Standard A5–*Number Theory Concepts*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify factors and multiples.

The items and student results reviewed by the task force did not warrant any specific observations about areas where students were unsuccessful.

Grade 5: Observations for Standard A5–*Number Theory Concepts*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify factors and multiples.

Students who are **unsuccessful** have the greatest difficulty with
- identifying prime numbers.

Grades 3–5: Implications for Instruction for Standard A5–*Number Theory Concepts*

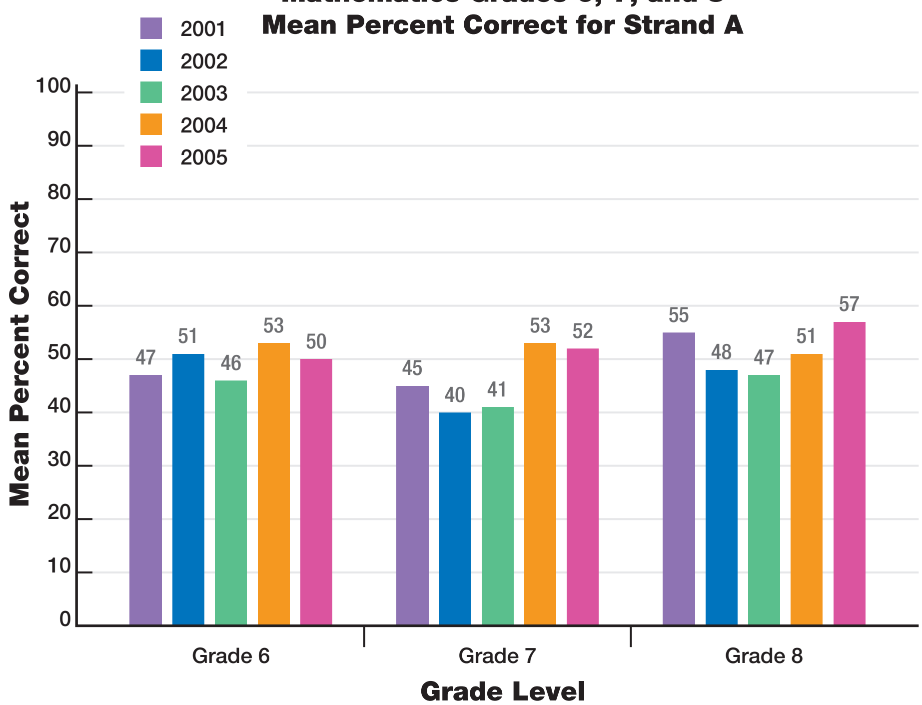
After reviewing student performance on items, the task force recommends instruction that emphasizes meaning, concept, and application, in addition to learning number facts. Students should practice by using arrays and base-10 blocks, physical models, and other manipulatives to understand the concept of multiplication. The goal of this practice is to reinforce factors and multiples and, at Grade 5, prime numbers.

Grades 6–8

Strand A Results for Grades 6–8

The following graph illustrates the performance of students in Grades 6–8 on Strand A (*Number Sense, Concepts, and Operations*).

Graph M-41
Mathematics Grades 6, 7, and 8
Mean Percent Correct for Strand A



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the Strand A (*Number Sense, Concepts, and Operations*) standards and benchmarks for Grades 6–8. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Strand A

Standard A1: The student understands the different ways numbers are represented and used in the real world.
Benchmark MA.A.1.3.1: The student associates verbal names, written word names, and standard numerals with integers, fractions, decimals; numbers expressed as percents; numbers with exponents; numbers in scientific notation; radicals; absolute value; and ratios. (Assessed with A.1.3.4)
Benchmark MA.A.1.3.2: The student understands the relative size of integers, fractions, and decimals; numbers expressed as percents; numbers with exponents; numbers in scientific notation; radicals; absolute value; and ratios.
Benchmark MA.A.1.3.3: The student understands concrete and symbolic representations of rational numbers and irrational numbers in real-world situations. (Assessed with A.1.3.4 and D.2.3.1)
Benchmark MA.A.1.3.4: The student understands that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents, radicals, and absolute value. (Also assesses A.1.3.1 and A.1.3.3)

Grade 6: Observations for Standard A1–*Representing Numbers*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- order numbers when they are represented using the same format (e.g., when decimals have the same number of places after the decimal point); and
- select the least or greatest value in a set of rational numbers.

Students who are **unsuccessful** have the greatest difficulty with
- ordering decimals when the numbers have different numbers of digits following the decimal point; and
- ordering numbers when they are represented in different formats.

Grade 7: Observations for Standard A1–*Representing Numbers*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify relative size on a number line;
- complete simple computations with exponents; and
- select the least or greatest value in a set of rational numbers.

Students who are **unsuccessful** have the greatest difficulty with
- completing multiple-step problems or conversions in a variety of formats; and
- solving problems that use the term *not* when asking the student to find a value fitting a given description.

Grade 8: Observations for Standard A1–*Representing Numbers*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- select the least or greatest value in a set of rational numbers.

Students who are **unsuccessful** have the greatest difficulty with

- multiple-step problems; and
- converting between formats (e.g., scientific notation, radicals, and standard notation).

Grades 6–8: Implications for Instruction for Standard A1–*Representing Numbers*

For Grade 6, the task force recommends that teachers present varied formats when representing numbers, as well as more problems that require persistence and thought. Students should also be provided with more instruction on decimal place value and equivalent forms. For Grade 7, the task force recommends the use of more visuals (e.g., to teach relative size of numbers). For Grades 7 and 8, the task force recommends giving students more opportunities to work with multiple-step conversions, such as feet per minute to miles per hour.

Standard A2: The student understands number systems.
Benchmark MA.A.2.3.1: The student understands and uses exponential and scientific notation.
Benchmark MA.A.2.3.2: The student understands the structure of number systems other than the decimal number system. (Not assessed)

Grade 6: Observations for Standard A2–*Number Systems*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- perform calculations with an exponent of 2.

Students who are **unsuccessful** have the greatest difficulty with

- interpreting the use of a given number expressed in exponential form (e.g., 3^2 could mean the area of a square with a length of 3 units on each side).

Grade 7: Observations for Standard A2–*Number Systems*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific recommendations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with

- converting from scientific notation to standard form; and
- converting from word form to scientific notation.

Grade 8: Observations for Standard A2–*Number Systems*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific recommendations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with

- completing multiple-step, scientific notation problems.

Grades 6–8: Implications for Instruction for Standard A2–*Number Systems*

For Grade 6, the task force recommends giving students more opportunities to work with exponents. For Grades 7 and 8, students should be able to express numbers in scientific notation and apply scientific notation in a broad problem-solving context.

Standard A3: The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.
Benchmark MA.A.3.3.1: The student understands and explains the effects of addition, subtraction, multiplication, and division on whole numbers, fractions, including mixed numbers, and decimals, including the inverse relationships of positive and negative numbers.
Benchmark MA.A.3.3.2: The student selects the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers, ratios, proportions, and percents, including the appropriate application of the algebraic order of operations.
Benchmark MA.A.3.3.3: The student adds, subtracts, multiplies, and divides whole numbers, decimals, and fractions, including mixed numbers, to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator.

Grade 6: Observations for Standard A3–*Operations*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- use the distributive property;
- identify the expression that correctly models a real-world problem;
- use grouping symbols to accurately complete order of operations;
- compute simple percent problems; and
- perform one-step operations with decimals (see sample item below).

Students who are **unsuccessful** have the greatest difficulty with
- multiple-step operations with decimals;
- order of operations, including exponents; and
- order of operations without grouping symbols.

Which number, when multiplied by 0.99, gives a product between 10 and 11?

A. 9
B. 10
☞ C. 11
D. 12

☞ Correct answer

Most recent student results

24% chose option A
17% chose option B
48% chose option C
11% chose option D

Strand A

Grade 7: Observations for Standard A3–*Operations*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify the correct expression that models the real-world problem; and
- compute simple percent problems (see sample item below).

Students who are **unsuccessful** have the greatest difficulty with

- dividing by fractions and decimals; and
- multiple-step problems with fractions, decimals, and percents.

Directors at a day care center planned to build a playhouse at an estimated cost of $350. The actual cost was 20% more than the estimate. What was the actual cost to build the playhouse?

Correct answer: 420

Most recent student results

49% of Grade 7 students answered this problem correctly.

Grade 8: Observations for Standard A3–*Operations*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify the correct expression that models the real-world problem;
- identify extraneous information;
- complete order of operations with grouping symbols; and
- understand the effects of operations on numbers (see sample item below).

Students who are **unsuccessful** have the greatest difficulty with

- dividing mixed numbers;
- completing order of operations without grouping symbols; and
- completing multiple-step problems.

Which of the following numbers, when multiplied by itself, would give an answer greater than itself?

A. $\frac{4}{5}$

☞ B. $\frac{5}{3}$

C. 0.05

D. 0.7

☞ Correct answer

Most recent student results

15% chose option A
53% chose option B
13% chose option C
19% chose option D

Grades 6–8: Implications for Instruction for Standard A3–*Operations*

For Grade 6, the task force recommends additional practice and emphasis on problems with order of operations involving exponents, and multiple-step problems that include operations with decimals. For Grades 7 and 8, more emphasis should be placed on problem solving (i.e., translating words into numbers needed for computation), using a variety of strategies to analyze a word problem and create an accurate mathematical expression or equation, and using multiple skills within one problem.

Standard A4 : The student uses estimation in problem solving and computation.

Benchmark MA.A.4.3.1: The student uses estimation strategies to predict results and to check the reasonableness of results. (Also assesses A.4.2.1, B.2.3.1, and B.3.3.1)

Grade 6: Observations for Standard A4–*Estimation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- estimate the distance along a path consisting of two or more line segments; and
- visually estimate volume (e.g., the number of beans in a jar).

Students who are **unsuccessful** have the greatest difficulty with

- multiple-step problems involving estimation of individual parts to find a whole (e.g., the number of trees in a section used as a benchmark to find the total in a park); and
- applying estimation to a real-world model or a drawing of a real-world model.

Grade 7: Observations for Standard A4–*Estimation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- interpret tables (e.g., given a table containing expected ranges of animals' lifespans, tell how many times greater one animal's lifespan might be as compared to another's).

Students who are **unsuccessful** have the greatest difficulty with

- estimating distance on a graphic along a path that is not made up of line segments (e.g., curved lines).

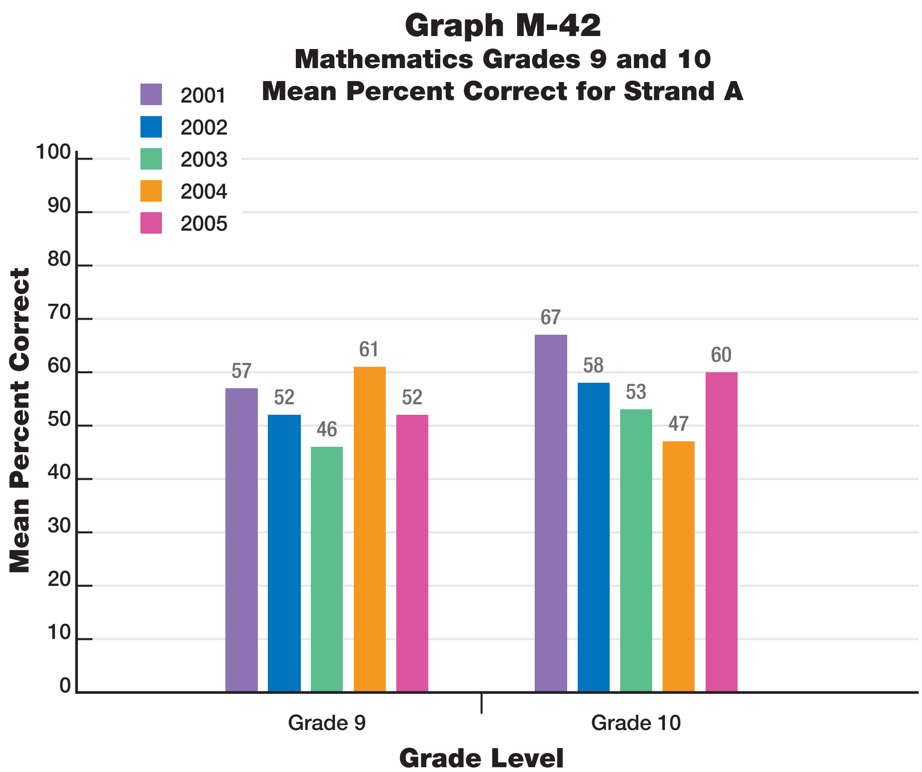
Grades 6–8: Implications for Instruction for Standard A4–*Estimation*

The task force recommends that students are provided with more opportunities to estimate in the context of real-life situations. The situations should involve measures such as distances or volumes and students should justify their estimates using a variety of representations.

Grades 9–10

Strand A Results for Grades 9–10

The following graph illustrates the performance of students in Grades 9–10 on Strand A (*Number Sense, Concepts, and Operations*).



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Number Sense, Concepts, and Operations* standards and benchmarks for Grades 9–10. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Strand A

Standard A1: The student understands the different ways numbers are represented and used in the real world.
Benchmark MA.A.1.4.1: The student associates verbal names, written word names, and standard numerals with integers, rational numbers, irrational numbers, real numbers, and complex numbers. (Assessed with A.1.4.4)
Benchmark MA.A.1.4.2: The student understands the relative size of integers, rational numbers, irrational numbers, and real numbers.
Benchmark MA.A.1.4.3: The student understands concrete and symbolic representations of real and complex numbers in real-world situations. (Assessed with A.1.4.4)
Benchmark MA.A.1.4.4: The student understands that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents, radicals, absolute value, and logarithms. (Also assesses A.1.4.1 and A.1.4.3)

Grade 9: Observations for Standard A1–*Representing Numbers*

Analysis of student performance data reveals the following:

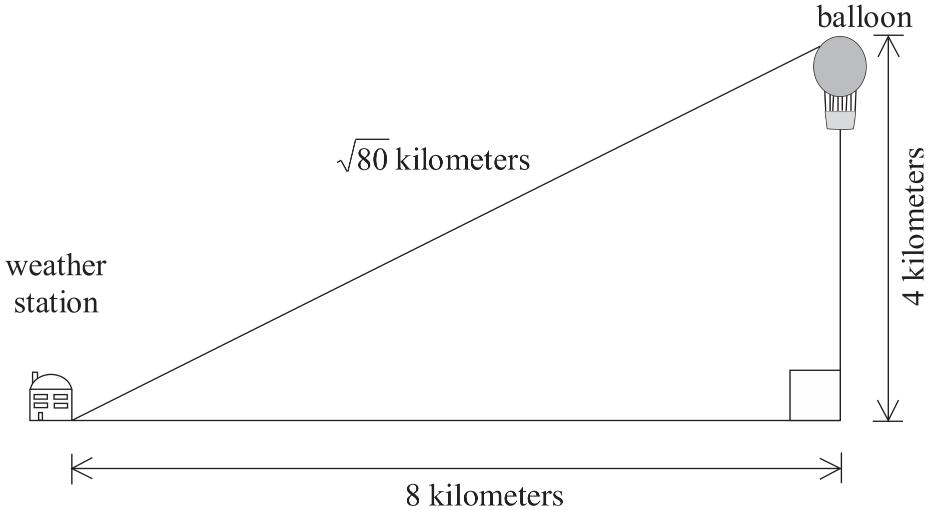
Students who are **successful** are able to

- solve routine one- and two-step problems involving basic computations;
- recognize equivalent forms of numbers (see sample item on the following page);
- convert radicals to decimals and convert scientific notation to standard notation; and
- recognize or find equivalent forms using scientific notation.

Students who are **unsuccessful** have the greatest difficulty with

- scientific notation and/or radicals (relative to numbers in standard form); and
- conversion of scientific notation and/or radicals for use in problem solving.

A meteorologist launched a weather balloon 8 kilometers from the weather station. After rising to a height of 4 kilometers, the balloon was $\sqrt{80}$ kilometers from the station.



Which of the following is equivalent to $\sqrt{80}$?

☞ A. $4\sqrt{5}$
B. $4\sqrt{20}$
C. $8\sqrt{10}$
D. $16\sqrt{5}$

☞ Correct answer

Most recent student results

50% chose option A
16% chose option B
26% chose option C
7% chose option D

Grade 10: Observations for Standard A1–*Representing Numbers*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- understand relative size of numbers;
- compare numbers in the same and different formats; and
- understand and compute with numbers in scientific notation.

Students who are **unsuccessful** have the greatest difficulty with

- applying skills in this standard beyond the routine or straightforward computation (i.e., the skill placed in context causes confusion); and
- identifying expressions involving the manipulation of percents (e.g., given that there is a 30% discount on x, find the final cost).

Grades 9–10: Implications for Instruction for Standard A1–*Representing Numbers*

For Grade 9, the task force recommends that students are provided with more practice using equivalent forms of a number (e.g., radicals or scientific notation) to solve a problem. This includes using the equivalent form beyond just the conversion to find an answer. The introduction of simplifying radicals, with emphasis on perfect squares, should occur before Grade 10.

For Grade 10, students should continue working with relative size of numbers. There has been improvement in this skill, but more fluency is needed. Increased practice on the understanding and application of percents is needed in problem-solving situations, including writing expressions involving manipulation of the percents when subtraction from or addition to 100% is necessary.

Standard A3: The student understands the effects of operations on numbers and the relationships among these operations, selects appropriate operations, and computes for problem solving.
Benchmark MA.A.3.4.1: The student understands and explains the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships. (Also assesses A.2.4.2)
Benchmark MA.A.3.4.2: The student selects and justifies alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (Also assesses A.2.4.2. and A.3.3.2)

Grade 9: Observations for Standard A3–*Operations*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- compute and translate with whole numbers (i.e., not with integers).

Students who are **unsuccessful** have the greatest difficulty with

- performing basic operations with integers and rational numbers; and
- manipulating and simplifying literal expressions and solving literal equations, which is solving for a variable in terms of one or more other variables (e.g., given $wx + wz = y$, solve for z).

Grade 10: Observations for Standard A3–*Operations*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- complete problems with routine order of operations (however, as order of operations become more complex, students are less successful);
- complete straightforward one- and two-step problems; and
- understand and apply proportion concepts.

Students who are **unsuccessful** have the greatest difficulty with

- rules applied to exponents;
- extracting details from diagrams and text and using these together to solve a problem;
- determining percent of increase and decrease, particularly when the amount is greater than 100 percent (see sample item on the following page); and
- manipulating and simplifying literal expressions and solving literal equations, which is solving for a variable in terms of one or more other variables (e.g., given $\frac{w}{x} - \frac{w}{y} = z$, solve for x).

Strand A

Henry Ford's world land-speed record of 91.37 miles per hour (mph) lasted less than a month after he set it in 1904. In 1997 the world land-speed record of 763.04 mph was the first to break the sound barrier. What was the **percent of increase** of the 1997 land-speed record over Henry Ford's 1904 land-speed record?

Correct answer: 735

Most recent student results

4% of Grade 10 students answered this problem correctly.

Grades 9–10: Implications for Instruction for Standard A3–*Operations*

For Grade 9, the task force recommends that students broaden their experience with applying skills in this standard. Practice is needed in solving equations for a variable in terms of one or more other variables. Teachers should encourage students to sketch pictures or diagrams if there is not one provided. Students should have more exposure to and work with percents larger than 100%.

For Grade 10, more emphasis should be placed on working with fractional equations and solving equations for a variable in terms of one or more other variables. Applying exponents and using them to solve problems should have increased emphasis in the classroom. Students at this level continue to struggle with percent of increase and percent of decrease, particularly when the percents are more than 100%. Teachers should continue to use word problems to provide opportunities for realistic applications of skills.

Standard A4: The student uses estimation in problem solving and computation.

Benchmark MA.A.4.4.1: The student uses estimation strategies in complex situations to predict results and to check the reasonableness of results. (Also assesses A.4.2.1 and B.3.4.1)

Grade 9: Observations for Standard A4–*Estimation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- determine simple estimations of linear distance.

Students who are **unsuccessful** have the greatest difficulty with

- estimating area and volume.

Grade 10: Observations for Standard A4–*Estimation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- estimate percents/portions in a circle graph.

Students who are **unsuccessful** have the greatest difficulty with

- estimating a measurement of a figure and then applying the estimation to further problem solving (e.g., finding the area of a figure, then multiplying the area by a price per square foot to arrive at a total cost).

Grades 9–10: Implications for Instruction for Standard A4–*Estimation*

For both Grades 9 and 10, the task force recommends that students be given more practice estimating the area and volume of irregular figures and using the estimation in problem-solving situations.

Strand B–*Measurement*

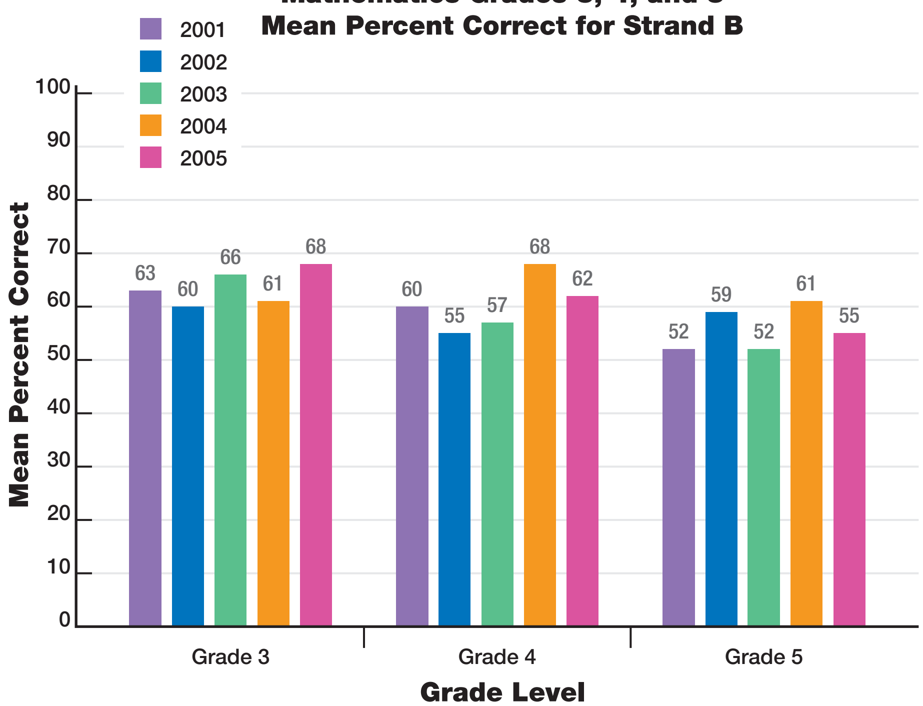
The current Sunshine State Standards have overlap in Strand B (*Measurement*) and Strand C (*Geometry and Spatial Sense*) in the treatment of geometric figures. As a result, items that assess benchmarks in each strand may seem quite similar within the same grade, with distinctions clarified in the *FCAT Mathematics Item Specifications*.

Grades 3–5

Strand B Results for Grades 3–5

The following graph provides the mean percent correct trends on Strand B (*Measurement*) for Grades 3–5.

Graph M-43
Mathematics Grades 3, 4, and 5
Mean Percent Correct for Strand B



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Measurement* standards and benchmarks for Grades 3–5. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Standard B1: The student measures quantities in the real world and uses the measures to solve problems.
Benchmark MA.B.1.2.1: The student uses concrete and graphic models to develop procedures for solving problems related to measurement including length, weight/mass, time, temperature, perimeter, area, volume/capacity and angle. (Also assesses C.2.2.1) (Grades 3–4: Not assessed; Grade 5: Assessed with C.2.2.1)
Benchmark MA.B.1.2.2: The student solves real-world problems involving length, weight, perimeter, area, capacity, volume, time, temperature, and angles.

Grade 3: Observations for Standard B1–*Measurement*

Analysis of student performance data reveals the following:

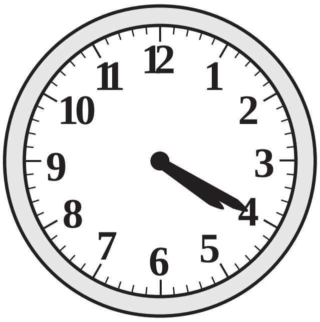
Students who are **successful** are able to

- use analog clocks to find the elapsed time (the number of hours) between two events (see sample item below); and
- determine perimeter when measurements are clearly labeled on a diagram, rather than determining the perimeter of a figure placed on a grid with a scale (e.g., length and width).

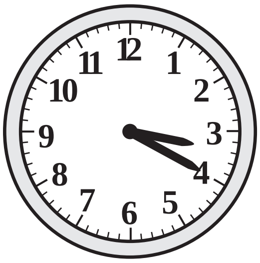
Students who are **unsuccessful** have the greatest difficulty with

- finding the starting time of an event, given the event's duration and ending time.

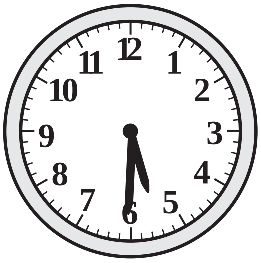
Nicole began taking a test at the time shown on the clock below.



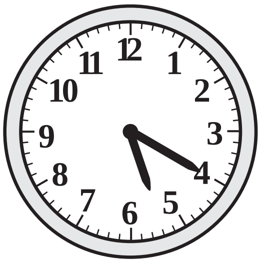
She finished the test in 1 hour. Which of the following shows the time Nicole finished the test?



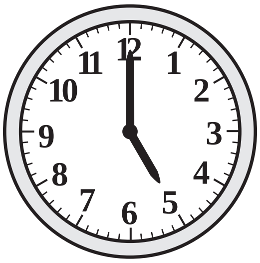
A.



B.



☞ C.



D.

☞ Correct answer

Most recent student results

4% chose option A
6% chose option B
70% chose option C
21% chose option D

Grade 4: Observations for Standard B1–*Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- calculate elapsed time using analog clocks with shorter time frames (e.g., 1 to 2 hours versus $10\frac{1}{2}$ hours); and
- determine perimeter when measurements on a diagram are provided.

Students who are **unsuccessful** have the greatest difficulty with
- calculating time when the problem involves longer time intervals.

Grade 5: Observations for Standard B1–*Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify angles (i.e., acute, obtuse, right, straight) and angle measurements;
- calculate elapsed time with hours and minutes; and
- calculate perimeter, area, and volume when measurements are provided on diagrams.

The items and student results reviewed by the task force did not warrant any specific observations about areas where students were unsuccessful.

Grades 3–5: Implications for Instruction for Standard B1–*Measurement*

The task force recommends that students in Grades 3 and 4 use analog clocks to practice solving elapsed-time problems that go both backward and forward in time. In addition, students should practice problems that require using time intervals greater than two hours. Students in Grades 3–5 should continue to practice finding length, weight, mass, capacity, and volume through hands-on activities using real-world objects.

Standard B2: The student compares, contrasts, and converts within systems of measurement (both standard/nonstandard and metric/customary).
Benchmark MA.B.2.2.1: The student uses direct (measured) and indirect (not measured) measures to calculate and compare measurable characteristics.
Benchmark MA.B.2.2.2: The student selects and uses appropriate standard and nonstandard units of measurement, according to type and size. (Also assesses B.4.2.1)

Grade 3: Observations for Standard B2–*Systems of Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- use pictures to identify and select appropriate units for measuring.

Students who are **unsuccessful** have the greatest difficulty with

- understanding the relative distance of a kilometer.

Grade 4: Observations for Standard B2–*Systems of Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- convert from one metric unit to another metric unit; and
- identify and select appropriate units for measuring.

Students who are **unsuccessful** have the greatest difficulty with

- comprehending the length of kilometers; and
- solving multiple-step word problems involving more than one unit of measurement (e.g., 3 pounds 2 ounces or 16 yards 2 feet). (See sample item on the following page.)

On the first visit to the veterinarian, Eleanor's cat weighed 3 pounds 4 ounces. On the second visit, her cat weighed 4 pounds 2 ounces. How much weight, **in ounces**, did Eleanor's cat gain between the first and second visit to the veterinarian?

16 ounces = 1 pound

A. 0.8
B. 1
C. 7.6
☞ D. 14

☞ Correct answer

Most recent student results

20% chose option A
18% chose option B
23% chose option C
39% chose option D

Grade 5: Observations for Standard B2–*Systems of Measurement*

Analysis of student performance data reveals the following:

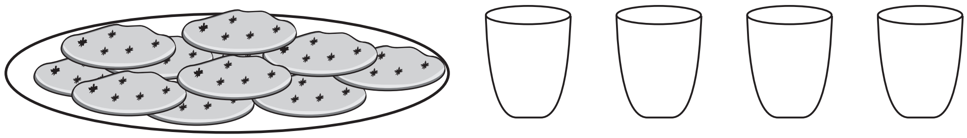
Students who are **successful** are able to

- convert time, including fractional units of time; and
- identify the meter as an appropriate unit of length for measuring things such as the dimensions of rooms.

Students who are **unsuccessful** have the greatest difficulty with

- solving problems with kilometer units; and
- performing conversions within multiple-step problems (see sample item on the following page).

Anita and Dan are serving cookies and juice to their class.



1 gallon = 128 ounces

How many 8 ounce glasses can they fill with 2 gallons of juice?

Correct answer: 32

Most recent student results

43% of Grade 5 students answered this problem correctly.

Grades 3–5: Implications for Instruction for Standard B2–*Systems of Measurement*

The task force recommends that students be given more opportunities to develop an awareness of the length of a kilometer and acquire additional practice performing conversions within the customary system of measurement, which includes conversions in multiple-step problems.

Standard B4: The student selects and uses appropriate units and instruments for measurement to achieve the degree of precision and accuracy required in real-world situations.

Benchmark MA.B.4.2.1: The student determines which units of measurement, such as seconds, square inches, dollars per tankful, to use with answers to real-world problems. (Assessed with B.2.2.2)

Benchmark MA.B.4.2.2: The student selects and uses appropriate instruments and technology, including scales, rulers, thermometers, measuring cups, protractors, and gauges, to measure in real-world situations. (Grade 5: Not assessed)

Grade 3: Observations for Standard B4–*Units of Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- read measurements on thermometers, rulers with centimeters and inches, and measuring cups.

The items and student results reviewed by the task force did not warrant any specific observations about areas where students were unsuccessful.

Grade 4: Observations for Standard B4–*Units of Measurement*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific observations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with
- identifying time to the nearest minute on an analog clock, especially when the time is later than half-past the hour (e.g., 6:52 or 3:46); and
- determining a length measurement when neither endpoint is aligned with the zero on a ruler.

Grades 3–5: Implications for Instruction for Standard B4–*Units of Measurement*

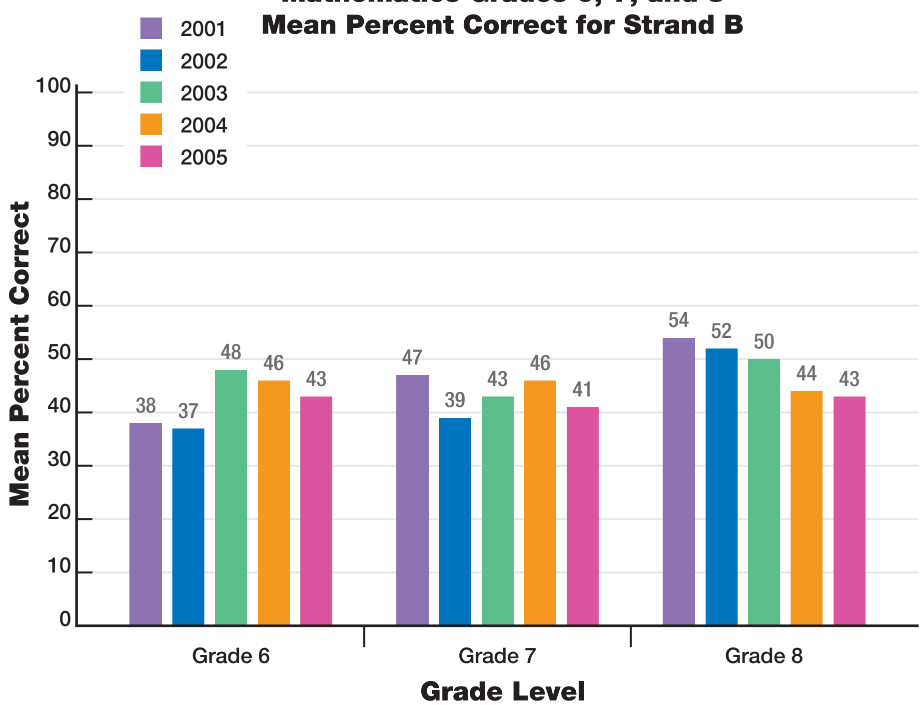
For Grade 4, the task force recommends that students practice telling time to the minute and practice determining length measurements when neither endpoint is aligned with the zero on a ruler.

Grades 6–8

Strand B Results for Grades 6–8

The following graph illustrates the performance of students in Grades 6–8 on the same strand (*Measurement*).

Graph M-44
Mathematics Grades 6, 7, and 8
Mean Percent Correct for Strand B



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Measurement* standards and benchmarks for Grades 6–8. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Standard B1: The student measures quantities in the real world and uses the measures to solve problems.
Benchmark MA.B.1.3.1: The student uses concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids and cylinders. (Also assesses B.1.2.2 and B.2.3.1)
Benchmark MA.B.1.3.2: The student uses concrete and graphic models to derive formulas for finding rates, distance, time, and angle measures. (Grade 6: Assessed with C.1.3.1; Grades 7–8: Also assesses B.1.2.2 and B.2.3.1)
Benchmark MA.B.1.3.3: The student understands and describes how the change of a figure in such dimensions as length, width, height, or radius affects its other measurements such as perimeter, area, surface area, and volume. (Also assesses C.2.3.1)
Benchmark MA.B.1.3.4: The student constructs, interprets, and uses scale drawings such as those based on number lines and maps to solve real-world problems. (Also assesses B.2.3.1)

Grade 6: Observations for Standard B1–*Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- complete one-step problems;
- identify measures of acute, right, straight, and obtuse angles; and
- determine measurements related to scale drawings.

Students who are **unsuccessful** have the greatest difficulty with
- finding perimeter or area if all dimensions are not labeled;
- finding perimeter and area of irregular shapes;
- elapsed time problems;
- finding differences of two volumes;
- applying properties of circles and triangles;
- applying the properties of triangles to solve a problem (e.g., given the vertex angle of an isosceles triangle, find each base angle);
- naming triangles using the vertices;
- multiple-step area and perimeter problems; and
- determining the effects on area and perimeter when a parameter is changed (see sample item on the following page).

Jerome had a rectangular garden that was 12 feet long and 14 feet wide. He expanded his garden by adding 3 feet to the length and 3 feet to the width. What is the perimeter of Jerome's new garden?

A. 32 feet
B. 58 feet
☞ C. 64 feet
D. 174 feet

☞ Correct answer

Most recent student results

43% chose option A
8% chose option B
42% chose option C
8% chose option D

Grade 7: Observations for Standard B1–*Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- perform calculations using a formula when all dimensions are given;
- understand the effects of changing one parameter of a two-dimensional figure; and
- interpret scale drawings when factors are whole numbers or when the scale includes only one fraction.

Students who are **unsuccessful** have the greatest difficulty with

- circular area problems;
- multiple-step problems;
- solving problems requiring a missing dimension to be calculated or solving problems requiring the student to work backward (e.g., finding the radius of a circle given the circumference);
- finding areas of trapezoids;
- multiple-step rate problems (see sample item on the following page);
- defining geometric terms and properties (especially complementary and supplementary angles);
- understanding how changes in parameters affect area;
- calculating volume; and
- interpreting scale drawings involving one fraction in proportion to another fraction (i.e., comparison of two fractions).

Maria began hiking on a trail at a rate of 4 miles per hour for 30 minutes. For the next 1 hour and 15 minutes, she hiked at a rate of 3 miles per hour and completed the trail. What is the total distance Maria hiked?

A. 3.50 miles
B. 3.75 miles
☞ C. 5.75 miles
D. 7.00 miles

☞ Correct answer

Most recent student results

12% chose option A
18% chose option B
43% chose option C
27% chose option D

Strand B

Grade 8: Observations for Standard B1—*Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- find the perimeter in one unit and convert to another unit (e.g., feet to miles);
- find the circumference of a circle, given its diameter;
- complete rate problems that require one or more steps when charts are provided;
- understand how changes in one dimension affects the area; and
- interpret scale drawings when factors are whole numbers or when the scale includes only one fraction.

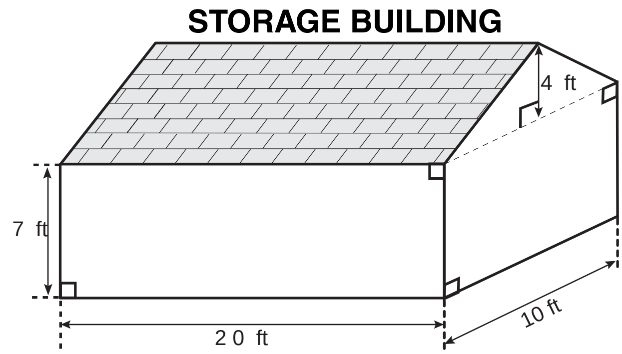
Students who are **unsuccessful** have the greatest difficulty with

- understanding the effects of increasing dimensions on two or more sides of a figure as it relates to area or perimeter;
- calculating the volume of a cylinder;
- solving multiple-step surface-area problems (see sample item on the following page);
- finding the surface area of a shape composed of smaller geometric shapes;
- multiple-step rate problems; and
- understanding the effect on volume when one dimension changes.

The following is a sample of a short-response task and a top-score response.

THINK SOLVE EXPLAIN

Paolo and Fred need to paint the 4 outside faces of a storage building. Before they can purchase the paint, they must calculate the surface area of the faces to be painted. A diagram of the building with 2 outside faces showing is given below.



Assuming that opposite sides of the building are congruent to each other, what is the total outside surface area, in square feet, of the 4 faces to be painted? Show all work necessary to justify your answer.

An explanation similar to the following:

Surface area of the front and back faces:

$2(20 \times 7) = 280$

Surface area of the two end faces:

$2(10 \times 7) = 140$

Surface area of the two triangular sections:

$2\left(\frac{1}{2}\right)(10 \times 4) = 40$

$280 + 140 + 40 = 460$

Total Surface Area of Outside Faces **460 square feet**

Strand B

Most recent student results

8% earned 2 points
15% earned 1 point
77% earned 0 points

Grades 6–8: Implications for Instruction for Standard B1–*Measurement*

For Grade 6, the task force emphasizes the need for students to draw and label figures when solving area or perimeter problems. Students should also work on multiple-step problems that involve elapsed time. There should also be opportunity for students to solve problems involving geometric figures. Students need additional experience in working with complex geometric shapes and in drawing conclusions to determine properties of angles, area, and perimeter of polygons. They should also be expected to apply the results of these conclusions.

For Grade 7, teachers should make sure that students work with irregular shapes (e.g., composite figures) to find area and perimeter. Students should also practice finding volume, work with two-step rate problems, and/or work with scale drawings involving multiple fractions in either the scale or in the dimensions.

For Grade 8, teachers should make sure that students work with irregular shapes (e.g., composite figures) to find area and/or perimeter. Students should also practice drawing diagrams to solve problems, finding surface area of composite figures, and finding surface area and volume of actual three-dimensional objects, including the effects of changes in dimensions. Such experience will help students develop visualization skills and allow them to understand the origin and application of formulas.

Standard B2: The student compares, contrasts, and converts within systems of measurement (both standard/nonstandard and metric/customary).
Benchmark MA.B.2.3.1: The student uses direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (Assessed with A.4.3.1, B.1.3.1, B.1.3.2, and B.1.3.4)
Benchmark MA.B.2.3.2: The student solves problems involving units of measure and converts answers to a larger or smaller unit within either the metric or customary system.

Grade 6: Observations for Standard B2–*Systems of Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- complete one-step unit conversions.

Students who are **unsuccessful** have the greatest difficulty with
- finding the difference or sum of denominate numbers (e.g., 5 ft 3 in. – 4 ft 8 in.); and
- conversions in the metric system.

Grade 7: Observations for Standard B2–*Systems of Measurement*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific recommendations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with

- finding the difference or sum of denominate numbers (e.g., the difference, in pounds, between 13.2 tons and 7.8 tons); and
- conversions in the metric system.

Grade 8: Observations for Standard B2–*Systems of Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- complete two-step length conversions using customary units.

Students who are **unsuccessful** have the greatest difficulty with

- two-step capacity conversion problems (e.g., converting 15 pints to gallons); and
- conversions in the metric system involving more than one step (e.g., mm to km).

Grades 6–8: Implications for Instruction for Standard B2–*Systems of Measurement*

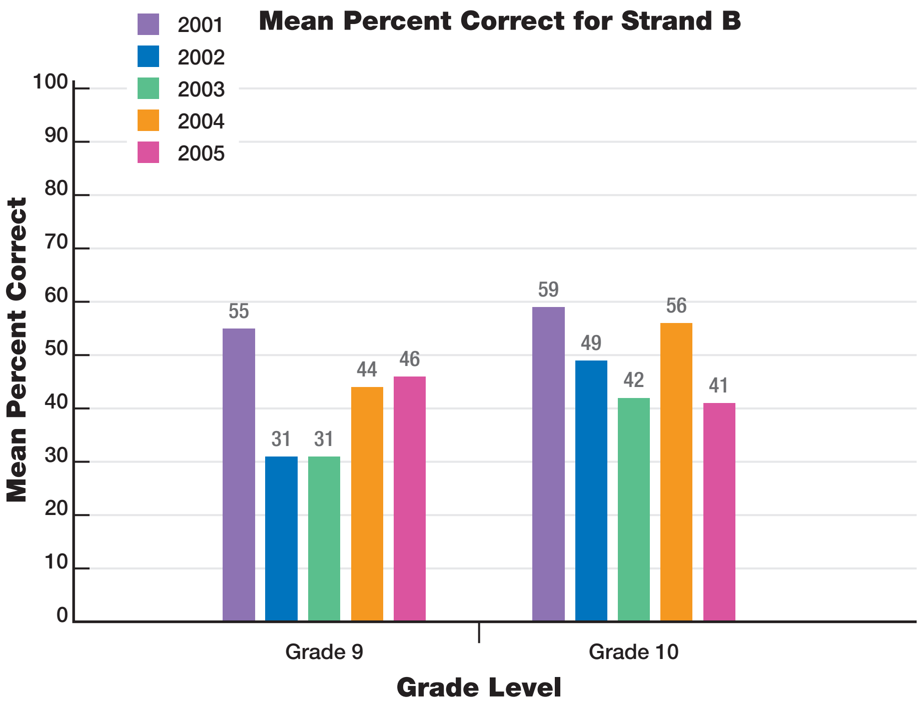
The task force recommends that students practice solving problems that involve multiple conversion steps and additional operations. This includes conversions within the metric system. Students need experience with manipulatives to make the connection between paper-and-pencil conversions and tangible unit conversions. Grades 6–7 students need practice working with differences or sums of denominate numbers.

Grades 9–10

Strand B Results for Grades 9–10

The following graph illustrates the performance of students in Grades 9–10 on the same strand (*Measurement*).

Graph M-45
Mathematics Grades 9 and 10
Mean Percent Correct for Strand B



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Measurement* standards and benchmarks for Grades 9–10. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Strand B

Standard B1: The student measures quantities in the real world and uses the measures to solve problems.
Benchmark MA.B.1.4.1: The student uses concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids, cones, and pyramids. (Also assesses B.1.2.2 and B.1.4.2)
Benchmark MA.B.1.4.2: The student uses concrete and graphic models to derive formulas for finding rate, distance, time, angle measures, and arc lengths. (Also assesses B.1.2.2)
Benchmark MA.B.1.4.3: The student relates the concepts of measurement to similarity and proportionality in real-world situations. (Grade 10: Assessed with C.2.4.1)

Grade 9: Observations for Standard B1–*Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- complete straightforward measurement problems (e.g., finding the area of a figure by applying a given formula).

Students who are **unsuccessful** have the greatest difficulty with

- basic geometry concepts that should have been mastered at previous grades (e.g., properties of polygons);
- calculating the volume, area, and perimeter of composite figures;
- solving a problem and then converting the units (e.g., calculating the volume of a figure in cubic feet and then converting to cubic yards); and
- converting rates using dimensional analyses (e.g., converting feet per second into miles per hour) and solving rate problems beyond a direct application of the distance formula (see sample item below).

Tremayne leaves his house each weekday morning at 7:38 a.m. and walks 1.5 miles to school. After arriving at school, he needs 3 minutes to get his belongings from his locker and walk to class. If his first class starts at 8:05 a.m., what is the **slowest** average rate of speed, **in miles per hour,** he can walk from his home to school in order to get to class on time?

Correct answer: 3.75

Most recent student results

5% of Grade 9 students answered this problem correctly.

Grade 10: Observations for Standard B1–*Measurement*

Analysis of student performance data reveals the following:

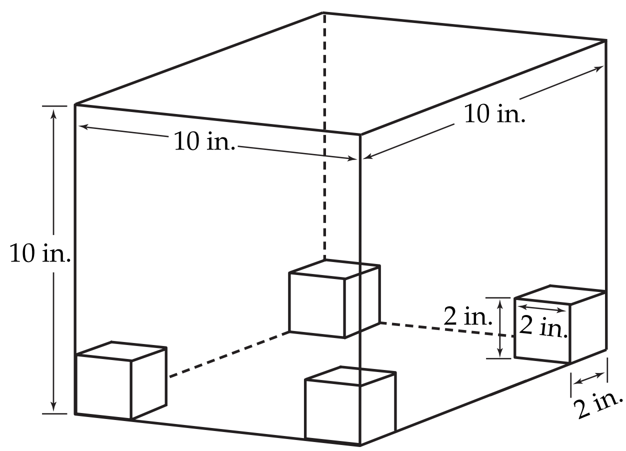
Students who are **successful** are able to

- complete one- and two-step problems that are straightforward.

Students who are **unsuccessful** have the greatest difficulty with

- basic geometry concepts that should have been mastered at previous grades, such as properties of polygons;
- multiple-step problems (see sample item below);
- solving a problem involving area or volume and then converting the units (e.g., calculating the volume of a figure in cubic feet and then converting to cubic yards);
- converting rates using dimensional analyses (e.g., converting feet per second into miles per hour) and solving rate problems beyond a direct application of the distance formula; and
- arc length and arc measures.

A shipping carton for computer parts is in the shape of a cube that measures 10 inches on each edge. In each of its bottom corners, the carton has 1 foam cube. Each foam cube measures 2 inches on an edge, as shown in the diagram below.



What is the volume, in cubic inches, of the empty space in the shipping carton when the 4 foam cubes are inside the box?

Correct answer: 968

Most recent student results

42% of Grade 10 students answered this problem correctly.

Grades 9–10: Implications for Instruction for Standard B1–*Measurement*

For Grades 9 and 10, the task force recommends giving students more opportunities to build stronger foundations in geometric concepts through hands-on activities. Through these types of activities, students also gain knowledge in linking geometric concepts to abstract formulas (e.g., surface area of a right circular cylinder). This will also help them understand and show how the formula for a composite figure is developed, and give students practice using more than one formula to represent a figure.

Standard B2: The student compares, contrasts, and converts within systems of measurement (both standard/nonstandard and metric/customary).
Benchmark MA.B.2.4.1: The student selects and uses direct (measured) or indirect (not measured) methods of measurement as appropriate.
Benchmark MA.B.2.4.2: The student solves real-world problems involving rated measures (miles per hour, feet per second). (Also assesses B.2.3.2)

Grade 9: Observations for Standard B2–*Systems of Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- perform one-step calculations of direct and indirect measurement where no conversions are necessary; and
- solve basic rate problems.

Students who are **unsuccessful** have the greatest difficulty with

- converting linear measures to cubic measures as part of the process to compute volume;
- converting non-typical rates and using them in a problem (e.g., breaking down an hourly rate into a per unit production rate, like chairs made per hour); and
- solving multiple-step rate problems with several rate parameters, including average rates (e.g., finding the cost per unit when the cost of the first hundred items is $4 per unit and additional items are $2 per unit, or finding the average rate of speed for an automobile trip if part was driven at 50 miles per hour for 3 hours and the rest at 70 miles per hour for 2 hours).

Strand B

Grade 10: Observations for Standard B2–*Systems of Measurement*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- perform one-step calculations of direct and indirect measurement where no conversions are necessary; and
- complete basic rate problems.

Students who are **unsuccessful** have the greatest difficulty with
- converting linear measures to cubic measures as part of the process to compute volume;
- converting non-typical rates and using them in a problem (e.g., breaking down an hourly rate into a per unit production rate, such as the number of chairs made per hour); and
- solving multiple-step rate problems with several rate parameters, including average rates (e.g., finding the cost per unit when the cost of the first hundred items is \$4 per unit and additional items are \$2 per unit, or finding the average rate of speed for an automobile trip if part was driven at 50 miles per hour for 3 hours and the rest at 70 miles per hour for two hours).

Grades 9–10: Implications for Instruction for Standard B2–*Systems of Measurement*

It is important to note that the task force felt there was some improvement in Grade 10 on some topics with which Grade 9 students had difficulty. The task force recommends that students continue to be encouraged to draw pictures to help them solve problems.

Strand C–*Geometry and Spatial Sense*

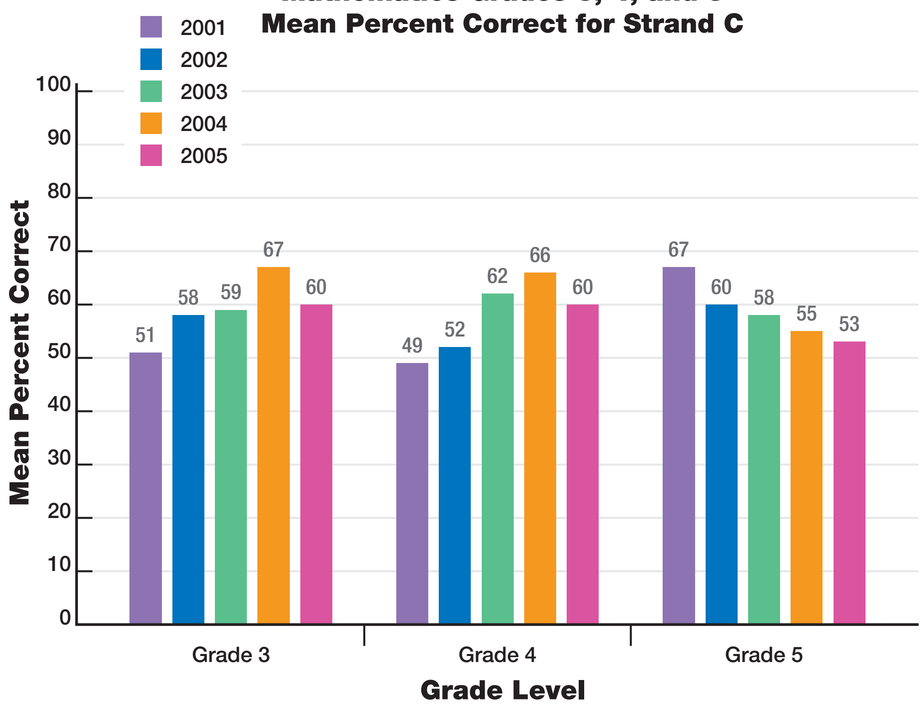
The current Sunshine State Standards overlap in Strand B (*Measurement*) and Strand C (*Geometry and Spatial Sense*) in the treatment of geometric figures. As a result, items that assess benchmarks in each strand may seem quite similar within the same grade, with distinctions clarified in the *FCAT Mathematics Item Specifications*.

Grades 3–5

Strand C Results for Grades 3–5

The following graph illustrates the performance of students in Grades 3–5 on Strand C (*Geometry and Spatial Sense*).

Graph M-46
Mathematics Grades 3, 4, and 5
Mean Percent Correct for Strand C



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Geometry and Spatial Sense* standards and benchmarks for Grades 3–5. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Standard C1: The student describes, draws, identifies, and analyzes two- and three-dimensional shapes.

Benchmark MA.C.1.2.1: The student, given a verbal description, draws and/or models two- and three-dimensional shapes and uses appropriate geometric vocabulary to write a description of a figure or a picture composed of geometric figures.

Grade 3: Observations for Standard C1–*Geometric Figures*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify right angles.

Students who are **unsuccessful** have the greatest difficulty with
- identifying the name of a polygon based on its number of sides, especially irregular polygons.

Grade 4: Observations for Standard C1–*Geometric Figures*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific observations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with
- understanding the meaning of *parallel* (e.g., students do not understand that parallel lines never intersect).

Grade 5: Observations for Standard C1–*Geometric Figures*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify the name of a polygon based on its number of sides, especially irregular polygons.

Students who are **unsuccessful** have the greatest difficulty with
- drawing a two-dimensional shape from a description involving multiple attributes.

Grades 3–5: Implications for Instruction for Standard C1–*Geometric Figures*

For Grade 3, the task force recommends teaching vocabulary and shapes at the same time, not in isolation. Students should practice identifying similarities and differences between shapes and practice working with polygons having up to six sides.

For Grade 4, teachers should also emphasize similarities and differences between shapes, types of angles, and polygons having up to eight sides. Students should practice with examples from everyday life to identify and reinforce geometric terms.

For Grade 5, students should be able to move beyond simply identifying shapes. Teachers should provide opportunities for students to create shapes based on verbal or written descriptions. These constructions can be created using manipulatives, such as straws, pipe cleaners, toothpicks, geo-boards, jump ropes, and rubber bands.

For Grades 3–5, teachers should create word walls for geometric terms used in mathematics. Teachers should provide students with experience in noting geometric shapes in the physical environment. Teachers should also connect geometric principles to art in the classroom and in special area classes.

Standard C2: The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.
Benchmark MA.C.2.2.1: The student understands the concepts of spatial relationships, symmetry, reflections, congruency, and similarity. (Also assesses B.1.2.1, B.1.2.2, C.1.2.1, and C.3.2.1)
Benchmark MA.C.2.2.2: The student predicts, illustrates, and verifies which figures could result from a flip (reflection), slide (translation), or turn (rotation) of a given figure.

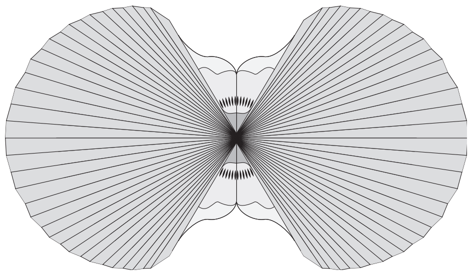
Grade 3: Observations for Standard C2–*Spatial Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- understand the concept of symmetry with figures that have one or two lines of symmetry.

Students who are **unsuccessful** have the greatest difficulty with
- understanding the concept of congruency (e.g., students tend to confuse congruency with similarity);
- visualizing 180-degree rotations (turns); and
- visualizing shapes being reflected (flipped).

The drawing shows an open clamshell.



How many lines of symmetry does the drawing have?

A. 3
☞ B. 2
C. 1
D. 0

☞ Correct answer

Most recent student results

14% chose option A
67% chose option B
14% chose option C
4% chose option D

Grade 4: Observations for Standard C2–*Spatial Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- understand the concept of symmetry (see sample item above).

Students who are **unsuccessful** have the greatest difficulty with
- visualizing 180-degree versus 90-degree rotations; and
- understanding the difference between congruency and similarity, particularly when the shapes are not oriented in the same direction.

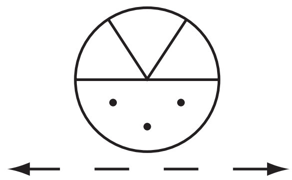
Grade 5: Observations for Standard C2–*Spatial Relationships*

Analysis of student performance data reveals the following:

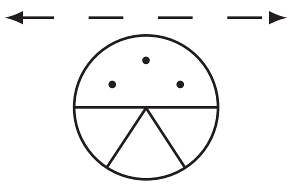
Students who are **successful** are able to
- identify lines of symmetry; and
- identify similar figures.

Students who are **unsuccessful** have the greatest difficulty with
- understanding the differences between 90-, 180-, and 270-degree rotations, both clockwise and counterclockwise;
- constructing similar figures; and
- performing more than one translation on a figure (see sample item on the following page).

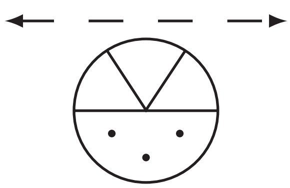
In art class, Rita drew the design below.



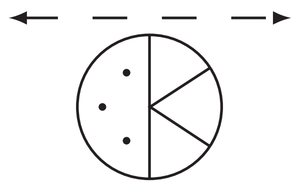
For her second design, she rotated the design 180 degrees and then flipped it over the dashed line. Which of the figures below represents Rita's second design?



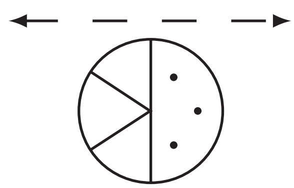
A.



☞ B.



C.



D.

☞ Correct answer

Most recent student results

38% chose option A
39% chose option B
12% chose option C
11% chose option D

Strand C

Grades 3–5: Implications for Instruction for Standard C2–*Spatial Relationships*

For Grades 3–5, teachers should use manipulatives to develop an understanding of reflections (flips), translations (slides), and rotations (turns). Teachers should avoid teaching 90-degree and 270-degree rotations in isolation. The goal is to develop a full understanding of the orientation of shapes in two dimensions (e.g., the same shape facing in different directions). One method that has proven effective has been the use of computer drawing programs to show rotations and reflections. Also, students need many and varied examples of symmetry, congruency, and/or similarity. Non-examples of these concepts must also be emphasized (e.g., Demonstrate the difference between a rectangle's lines of symmetry and its diagonals. A diagonal divides a rectangle into two equal-sized parts but is not a line of symmetry.).

Standard C3: The student uses coordinate geometry to locate objects in both two and three dimensions and to describe objects algebraically.
Benchmark MA.C.3.2.1: The student represents and applies a variety of strategies and geometric properties and formulas for two- and three-dimensional shapes to solve real-world and mathematical problems. (Also assesses C.2.2.1 and C.3.2.2)
Benchmark MA.C.3.2.2: The student identifies and plots positive ordered pairs (whole numbers) in a rectangular coordinate system (graph).

Grade 3: Observations for Standard C3–*Coordinate Geometry*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- calculate area on a grid;
- identify coordinates on a grid;
- calculate perimeter when measurements are given; and
- calculate perimeter of a simple figure when presented on a grid (see sample item on the following page).

Students who are **unsuccessful** have the greatest difficulty with
- finding the perimeter of a composite figure when presented on a grid; and
- understanding the difference between perimeter and area.

Grade 4: Observations for Standard C3–*Coordinate Geometry*

Analysis of student performance data reveals the following:

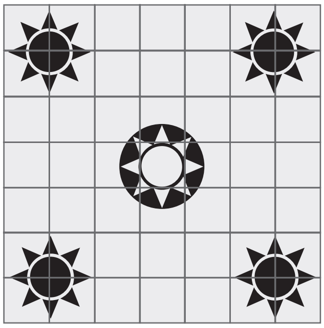
Students who are **successful** are able to

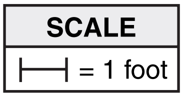
- calculate perimeter when measurements are given; and
- identify coordinates.

Students who are **unsuccessful** have the greatest difficulty with

- finding the perimeter in multiple-step problems; and
- finding the area or perimeter of composite or irregular shapes when some of the sides are not labeled.

Amelia is sewing a ribbon on the perimeter of her bedcover. The drawing below shows her bedcover.





What is the perimeter of the bedcover?

Perimeter = distance around an object

A. 49 feet
☞ B. 28 feet
C. 14 feet
D. 12 feet

☞ Correct answer

Most recent student results

26% chose option A
61% chose option B
6% chose option C
7% chose option D

Strand C

Grade 5: Observations for Standard C3–*Coordinate Geometry*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- complete single-step area and perimeter problems; and
- identify and locate ordered pairs on a coordinate grid.

Students who are **unsuccessful** have the greatest difficulty with

- finding area and perimeter in multiple-step problems (e.g., determining the areas of parts of composite shapes situated on grids);
- finding lengths of and working with composite shapes when not all sides are labeled;
- understanding irregular figures;
- identifying and drawing two-dimensional figures based on descriptions involving multiple attributes; and
- determining areas and perimeters on grids that do not have one-to-one scales.

Grades 3–5: Implications for Instruction for Standard C3–*Coordinate Geometry*

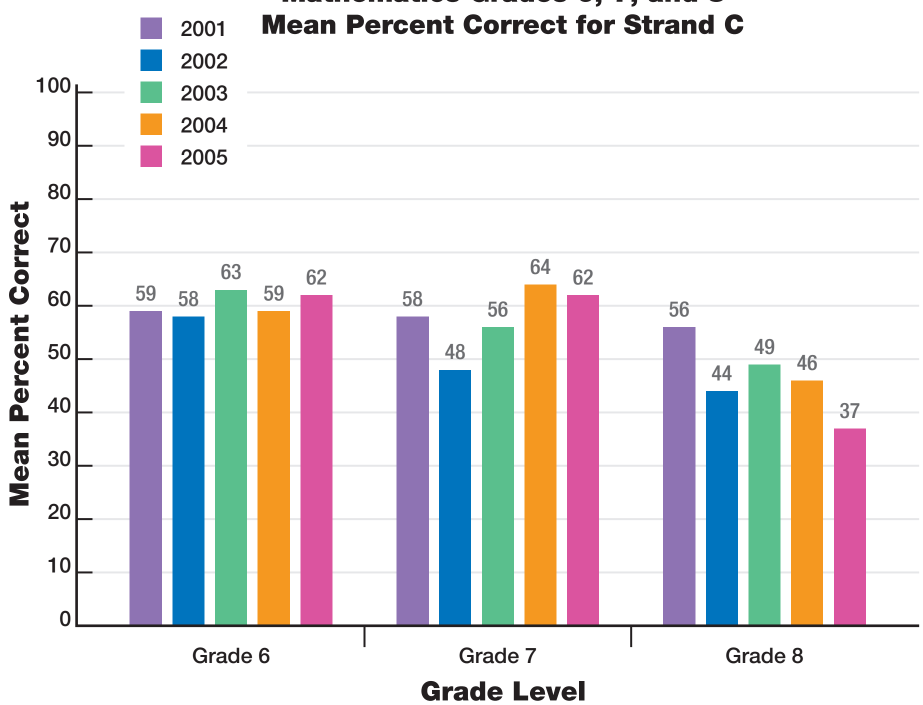
For Grades 3–5, the task force recommends that teachers emphasize geometric terminology and provide hands-on practice using blocks, colored tiles, and other manipulatives to compute the area and perimeter of regular, irregular, and, for Grades 4 and 5, composite figures. Instruction should include practice with figures drawn on grids that have scales to calculate the measurement of the sides of the figures. Teachers should find resources beyond the text by identifying and using real-world applications (e.g., painting a room) to represent multiple-step perimeter and area problems. Teachers should reinforce multiplication arrays to facilitate finding sides of a rectangle, given its area. Students should calculate both area and perimeter on the same figure and discuss the differences between them.

Grades 6–8

Strand C Results for Grades 6–8

The following graph illustrates the performance of students in Grades 6–8 on the same strand (*Geometry and Spatial Sense*).

Graph M-47
Mathematics Grades 6, 7, and 8
Mean Percent Correct for Strand C



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Geometry and Spatial Sense* standards and benchmarks for Grades 6–8. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Standard C1: The student describes, draws, identifies, and analyzes two- and three-dimensional shapes.
Benchmark MA.C.1.3.1: The student understands the basic properties of, and relationships pertaining to, regular and irregular geometric shapes in two and three dimensions. (Also assesses C.1.2.1)

Strand C

Grade 6: Observations for Standard C1–*Geometric Figures*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- recognize/identify parallel and perpendicular lines in a figure;
- identify relationships between line segments;
- understand the definitions of types of angles (e.g., right, acute, obtuse); and
- identify and distinguish between different types of polygons.

Students who are **unsuccessful** have the greatest difficulty with

- visualizing the nets of three-dimensional figures;
- distinguishing between the radius and diameter of a circle (i.e., students confuse the two concepts); and
- understanding the differences between regular and irregular polygons.

Grade 7: Observations for Standard C1–*Geometric Figures*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- visualize and identify two- and three-dimensional shapes;
- identify the relationship between line segments (e.g., perpendicular, parallel) in a real-world drawing without geometric notation;
- identify lines of symmetry; and
- understand the definitions of types of angles (e.g., right, acute, obtuse).

Students who are **unsuccessful** have the greatest difficulty with

- identifying perpendicular lines in a construction;
- identifying perpendicular lines using geometric notation; and
- identifying counterclockwise and clockwise rotations.

Grade 8: Observations for Standard C1–*Geometric Figures*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- determine interior angle measures in a polygon with some given information; and
- identify nets of three-dimensional figures.

Students who are **unsuccessful** have the greatest difficulty with

- angles that form 360 degrees; and
- identifying and classifying triangles (e.g., classifying scalene triangles).

Grades 6–8: Implications for Instruction for Standard C1–*Geometric Figures*

For Grade 6, the task force recommends that students be given opportunities to manipulate, draw, label, and construct geometric shapes. Students should experience working with the nets of three-dimensional figures. Students should also understand the formal symbolism and vocabulary of geometry.

For Grade 7, as in the previous grade, students should understand the formal symbolism and vocabulary of geometry. They should also be exposed more frequently to the terms *clockwise* and *counterclockwise* in their study of rotation.

For Grade 8, students need concrete experiences identifying triangles by type (e.g., scalene), as well as developing skills in identifying types of triangles based on information about sides and/or angles. Students should also use manipulatives to experiment with the sum of the measures of polygons.

Standard C2: The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.
Benchmark MA.C.2.3.1: The student understands the geometric concepts of symmetry, reflections, congruency, similarity, perpendicularity, parallelism, and transformations, including flips (reflections), slides (translations), turns (rotations), and enlargements. (Also assesses B.1.3.3, C.1.2.1, C.1.3.1, and C.3.3.1)
Benchmark MA.C.2.3.2: The student predicts and verifies patterns involving tessellations (a covering of a plane with congruent copies of the same pattern with no holes and no overlaps, like floor tiles). (Assessed with C.3.3.1)

Grade 6: Observations for Standard C2–*Spatial Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- translate a simple geometric figure in a coordinate plane and identify a new coordinate;
- identify lines of symmetry (see sample item on the following page);
- recognize reflections; and
- rotate a figure given explicit direction.

Students who are **unsuccessful** have the greatest difficulty with

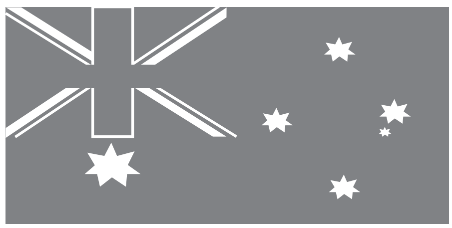
- rotating a figure without explicit direction.

National flags come in a variety of patterns.

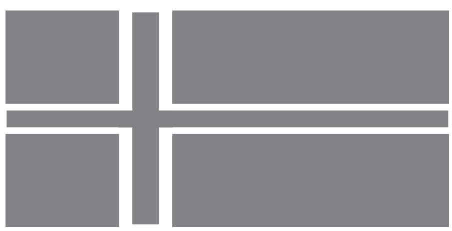
Which of these flags has the most lines of symmetry?

☞A. Switzerland

B. Australia



C. Iceland



D. Canada

☞ Correct answer

Most recent student results

60% chose option A
17% chose option B
13% chose option C
10% chose option D

Grade 7: Observations for Standard C2–*Spatial Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify lines of symmetry;
- name four-sided geometric figures; and
- recognize reflections.

Students who are **unsuccessful** have the greatest difficulty with

- rotational vocabulary (e.g., *clockwise* and *counterclockwise*);
- correctly translating all coordinates of a geometric figure in the coordinate plane;
- rotation of nontraditional figures (e.g., rotating clockwise around a vertex of a nontraditional figure); and
- identifying congruent corresponding parts of congruent figures.

Grade 8: Observations for Standard C2–*Spatial Relationships*

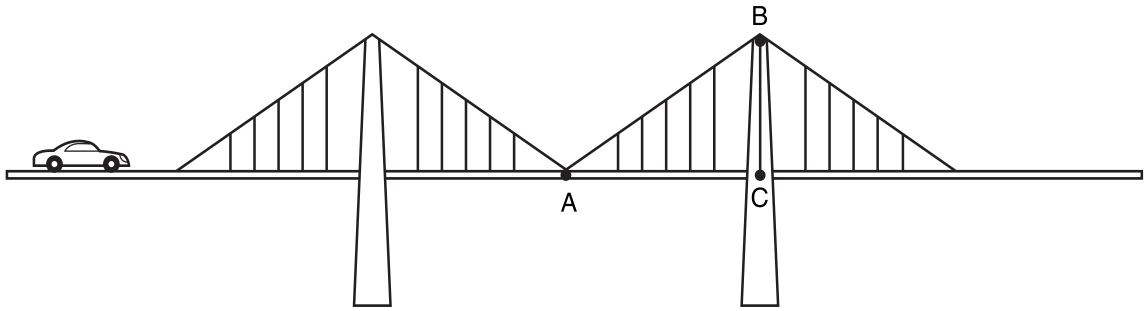
Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify parallel lines using geometric notation;
- identify a proportion given two similar triangles; and
- calculate the measure of the hypotenuse of a right triangle (see sample item on the following page).

The items and student results reviewed by the task force did not warrant any specific recommendations about areas where students were unsuccessful.

A side view of the suspension bridge on Franklin Street is shown below.



The length of the roadway from Point A to Point C is 90 feet. The vertical columns are perpendicular to the roadway, and the distance from Point C to Point B is 40 feet. Which measure is closest to the length of the steel cable connecting Point A to Point B?

A. 81 feet
B. 90 feet
☞ C. 98 feet
D. 130 feet

☞ Correct answer

Most recent student results

10% chose option A
15% chose option B
51% chose option C
24% chose option D

Strand C

Grades 6–8: Implications for Instruction for Standard C2–*Spatial Relationships*

For Grade 6, the task force recommends that students practice using the terms *clockwise* and *counterclockwise* when studying rotations.

For Grade 7, the task force recommends that students continue to practice using the terms *clockwise* and *counterclockwise* when studying rotations. Students should practice identifying the coordinates of transformed traditional figures (e.g., triangles, rectangles) as well as transformed nontraditional figures (e.g., puzzle pieces, irregular polygons) on a coordinate grid. More opportunities should be provided for students to identify the congruent corresponding parts of congruent figures.

For Grades 6–8, students should have more experience with transformations, especially rotations, on a coordinate plane. The task force also recommends that students use graph paper and/or technology to demonstrate multiple examples of transformations on a grid.

Standard C3: The student uses coordinate geometry to locate objects in both two and three dimensions and to describe objects algebraically.
Benchmark MA.C.3.3.1: The student represents and applies geometric properties and relationships to solve real-world and mathematical problems. (Also assesses C.2.3.1, C.2.3.2, and C.3.2.2)
Benchmark MA.C.3.3.2: The student identifies and plots ordered pairs in all four quadrants of a rectangular coordinate system (graph) and applies simple properties of lines.

Grade 6: Observations for Standard C3–*Coordinate Geometry*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify perpendicular lines in real-world situations;
- plot and identify coordinates; and
- identify similar figures.

The items and student results reviewed by the task force did not warrant any specific recommendations about areas where students were unsuccessful.

Grade 7: Observations for Standard C3–*Coordinate Geometry*

Analysis of student performance data reveals the following:

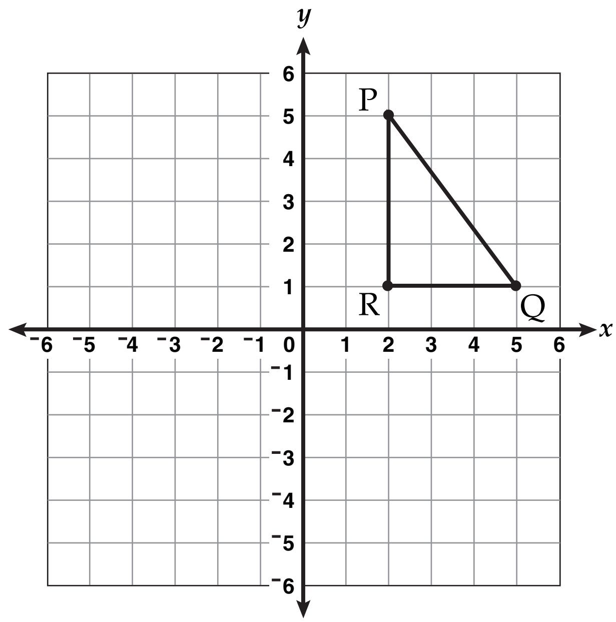
Students who are **successful** are able to

- identify corresponding sides of similar figures;
- identify reflections in the coordinate plane (see sample item on the following page); and
- understand similarity and identify similar triangles.

Students who are **unsuccessful** have the greatest difficulty with

- identifying coordinates of reflected figures;
- comparing descriptions of coordinates;
- identifying in which quadrant an ordered pair is located; and
- identifying the x- and y-intercepts in a given graph.

Triangle PQR is shown on the grid below. If △PQR is reflected (flipped) over the **x-axis**, what will be the coordinates of the vertices of the new triangle?



A. (2, -5), (-1, 5), (2, -1)
B. (2, -5), (5, 0), (2, -1)
C. (2, -5), (5, -1), (-1, 2)
☞ D. (2, -5), (5, -1), (2, -1)

☞ Correct answer

Most recent student results

16% chose option A
8% chose option B
18% chose option C
57% chose option D

Grade 8: Observations for Standard C3–*Coordinate Geometry*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify coordinates on a plane; and
- identify coordinates on a plane that satisfy the requirements to complete a regular polygon.

Students who are **unsuccessful** have the greatest difficulty with

- setting up and solving proportions from real-world examples involving similar triangles;
- rotating nontraditional figures about the origin in the coordinate plane;
- finding the slope of a given line drawn in the coordinate plane; and
- identifying relationships of line segments in geometric figures with formal geometry symbolism.

Grades 6–8: Implications for Instruction for Standard C3–*Coordinate Geometry*

For Grade 7, students need instruction in plotting points on a coordinate plane and more practice identifying the quadrant where given coordinates are located. Teachers should provide a variety of experiences requiring students to identify the *x*- and *y*-intercepts.

For Grade 8, students should have opportunities to practice rotating nontraditional figures and identifying coordinates of the vertices. Students need more experience in finding the slope of a given line drawn on the coordinate plane. Students also need more practice identifying perpendicular line segments on a coordinate grid.

For Grades 6–8, students need more experience setting up and solving proportions from problems presented in a real-world context.

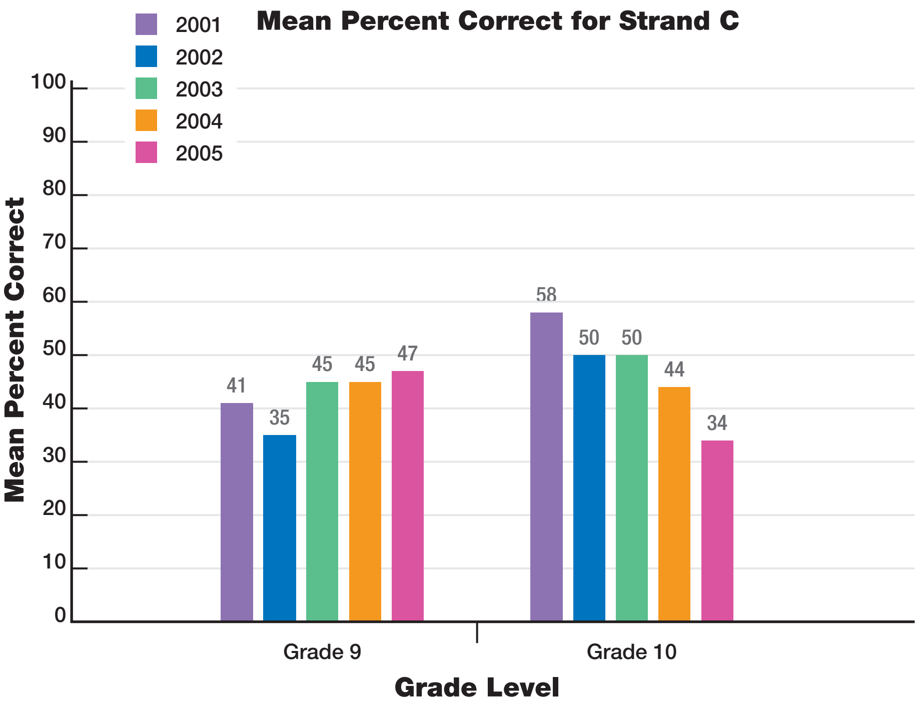
Strand C

Grades 9–10

Strand C Results for Grades 9–10

The following graph illustrates the performance of students in Grades 9–10 on the same strand (*Geometry and Spatial Sense*).

Graph M-48
Mathematics Grades 9 and 10
Mean Percent Correct for Strand C



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Geometry and Spatial Sense* standards and benchmarks for Grades 9–10. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Standard C1: The student describes, draws, identifies, and analyzes two- and three-dimensional shapes.
Benchmark MA.C.1.4.1: The student uses properties and relationships of geometric shapes to construct formal and informal proofs. (Also assesses C.1.2.1 and C.1.3.1)

Grade 9: Observations for Standard C1–*Geometric Figures*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify properties of polygons to which students were exposed in previous grades; and
- identify similar figures and solve problems involving similar triangles, given that the triangles have the same orientation.

Students who are **unsuccessful** have the greatest difficulty with
- understanding geometric terms (e.g., *isosceles, chord*);
- the defining characteristics of polygons (e.g., choosing the minimum requirements that ensure that a quadrilateral is defined as a rectangle);
- properties of polygons, including angle measurements in figures other than triangles and rectangles; and
- properties of circles.

Grade 10: Observations for Standard C1–*Geometric Figures*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify properties of figures with more than four sides;
- accurately apply the interior-angle sum formula; and
- identify properties of circles, including the concept of central angles and the manipulation of the 360 degrees.

Students who are **unsuccessful** have the greatest difficulty with
- understanding geometric terms (e.g., *chord*);
- using properties of circles and angles (e.g., diameter, chords) to solve problems;
- using properties of parallel lines and transversals to solve problems, especially when in a complex diagram; and
- using right triangles in multiple-step applications.

Grades 9–10: Implications for Instruction for Standard C1–*Geometric Figures*

The task force recommends that students practice expressing their thinking skills using correct mathematical terminology. Teachers should provide students with more experiences with polygons. Geometric concepts should be incorporated into the curriculum throughout Grade 9 to reinforce previous instruction. At both Grades 9 and 10, students should build on the informal reasoning learned in earlier grades to develop more formal reasoning.

Standard C2: The student visualizes and illustrates ways in which shapes can be combined, subdivided, and changed.
Benchmark MA.C.2.4.1: The student understands geometric concepts such as perpendicularity, parallelism, tangency, congruency, similarity, reflections, symmetry, and transformations including flips (reflections), slides (translations), turns (rotations), enlargements, rotations, and fractals. (Also assesses B.1.4.3, C.1.4.1, and C.3.4.1)
Benchmark MA.C.2.4.2: The student analyzes and applies geometric relationships involving planar cross-sections (the intersection of a plane and a three-dimensional figure). (Grade 9: Not assessed)

Grade 9: Observations for Standard C2–*Spatial Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- recognize transformations; and
- calculate interior and exterior angles of triangles.

Students who are **unsuccessful** have the greatest difficulty with

- identifying multiple congruent parts within a whole;
- identifying congruent shapes within a complex figure;
- using congruent or similar parts of two different figures to solve problems; and
- using similar triangles to solve problems when one triangle is embedded within the other.

Grade 10: Observations for Standard C2–*Spatial Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- solve problems involving similar triangles given basic representations (i.e., not embedded within one another nor oriented differently from one another);
- understand tessellations;
- recognize transformations;
- identify cross-sections of shapes; and
- calculate surface area and volume of three-dimensional figures, given a two-dimensional cross section representation and measurements required to perform the calculations.

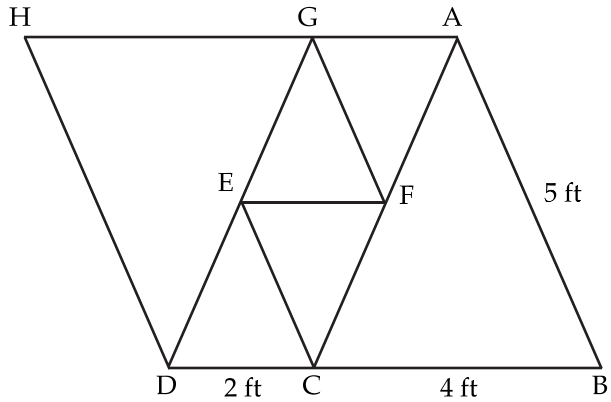
Students who are **unsuccessful** have the greatest difficulty with

- properties of cross sections (e.g., side lengths or area);
- using similar triangles to solve problems when one triangle is embedded within the other (see sample item on the following page);
- applications of isosceles trapezoids and isosceles triangles;
- computing cross sections of a three-dimensional figure, especially where the cross section is not parallel to a base; and
- applying the Pythagorean theorem in real-life situations.

The following is a sample of an extended-response task and a top-score response.

THINK
SOLVE
EXPLAIN

An architect is using **isosceles triangles** in the design of a bridge. In the diagram below, all line segments represent the steel beams needed to build this section of the bridge. Line segment $\overline{HA}$ is parallel to line segment $\overline{DB}$. ΔDEC is similar to ΔCAB and congruent to ΔAFG.



Part A Write and solve a proportion to determine the length, in feet, of line $\overline{EC}$. Show your work. *Work equivalent to the following:*

$$\frac{EC}{5} = \frac{2}{4}$$

$$EC = 5\left(\frac{2}{4}\right)$$

$$EC = \frac{10}{4}$$

$$EC = 2.5$$

Proportion $\frac{EC}{5} = \frac{2}{4}$ *OR other valid proportion for determining the length of $\overline{EC}$*

Length, in feet, of line $\overline{EC}$ 2.5 (feet)

Part B In the diagram, all the smaller triangles are congruent, and all the larger triangles are congruent. Determine the total length, in feet, of all the steel beams needed to build the section of the bridge shown. Show all of the work needed to determine the total length of the beams.

Work equivalent to the following:

Total length in feet = 6(2.5) + 2(5) + 3(2) + 2(4)

Total length in feet = 15 + 10 + 6 + 8

Total length in feet = 39

Total length in feet 39 (feet)

Most recent student results

17% earned 4 points
9% earned 3 points
24% earned 2 points
18% earned 1 point
32% earned 0 points

Strand C

Grades 9–10: Implications for Instruction for Standard C2–*Spatial Relationships*

As was the case in Standard C1, the task force recommends that students practice expressing their thinking using correct mathematical terminology. Teachers should provide students with more experiences with polygons. Geometric concepts should be incorporated into the curriculum throughout Grade 9 to reinforce previous instruction. At Grades 9–10, students should build on the informal reasoning learned in earlier grades to develop more formal reasoning.

Standard C3: The student uses coordinate geometry to locate objects in both two and three dimensions and to describe objects algebraically.
Benchmark MA.C.3.4.1: The student represents and applies geometric properties and relationships to solve real-world and mathematical problems including ratio, proportion, and properties of right triangle trigonometry. (Also assesses C.2.4.1)
Benchmark MA.C.3.4.2: The student using a rectangular coordinate system (graph), applies and algebraically verifies properties of two- and three-dimensional figures, including distance, midpoint, slope, parallelism, and perpendicularity. (Also assesses C.3.3.2 and D.2.4.1)

Grade 9: Observations for Standard C3–*Coordinate Geometry*

Analysis of student performance data reveals the following:

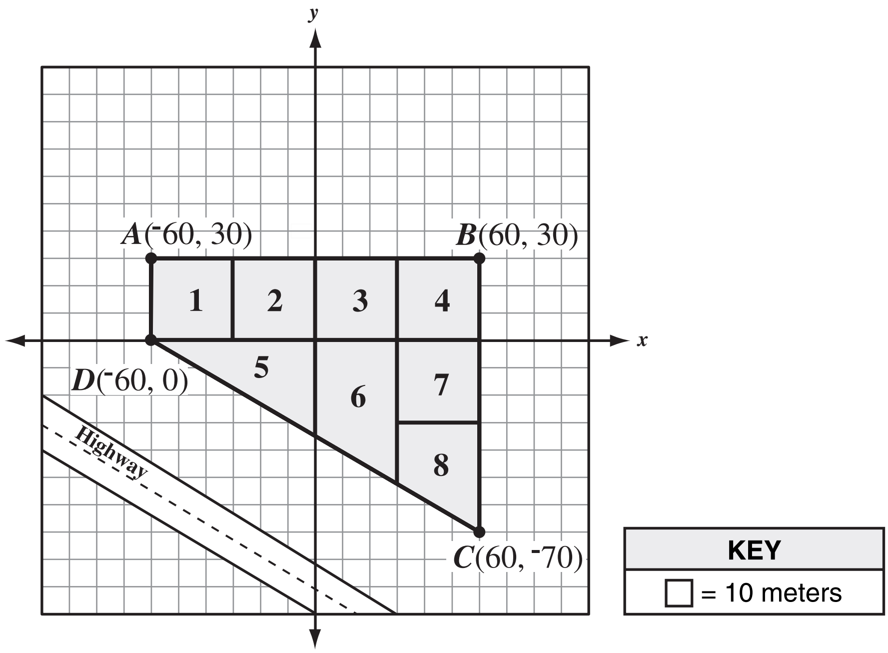
Students who are **successful** are able to

- complete transformations of geometric figures on a coordinate grid;
- compute the midpoint of a line segment;
- set up and compute straightforward scale and ratio problems; and
- apply the Pythagorean theorem in simple situations when solving for the hypotenuse as it is used in the distance formula.

Students who are **unsuccessful** have the greatest difficulty with

- applying the Pythagorean theorem when solving for the length of the leg of a right triangle as it is used in the distance formula;
- applying the distance formula in multiple-step problems (see sample item on the following page);
- understanding and calculating the slope of a line;
- problems involving 3-4-5 right triangles on a grid;
- naming coordinates of vertices for a reflection of a shape over a line that is not vertical or horizontal; and
- identifying equations of lines that are parallel or perpendicular to a given line.

Land developers are planning to subdivide a plot of land into 8 lots for a garden home development, as shown in the diagram below. They plan to build a rock wall along three sides of the property to block the highway noise, as indicated by $\overline{AD}$, $\overline{DC}$, and $\overline{CB}$.



Which of the following measures is closest to the length of the rock wall that will be built?

A. 389 meters
B. 286 meters
☞ C. 269 meters
D. 139 meters

☞ Correct answer

Most recent student results

17% chose option A
29% chose option B
42% chose option C
12% chose option D

Strand C

Grade 10: Observations for Standard C3–*Coordinate Geometry*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- compute the midpoint of a line segment;
- calculate slope;
- identify equations of lines that are parallel or perpendicular to a given line; and
- use the distance formula in a basic computational situation.

Students who are **unsuccessful** have the greatest difficulty with

- applying the Pythagorean theorem when solving for the length of the leg of a right triangle as it is used in the distance formula;
- applying the distance formula to solve a problem in a real-world situation; and
- identifying the coordinates of a vertex in a polygon by utilizing the characteristics of polygons (e.g., given the coordinates of three vertices in a rhombus, identify the coordinates of the fourth vertex).

Grades 9–10: Implications for Instruction for Standard C3–*Coordinate Geometry*

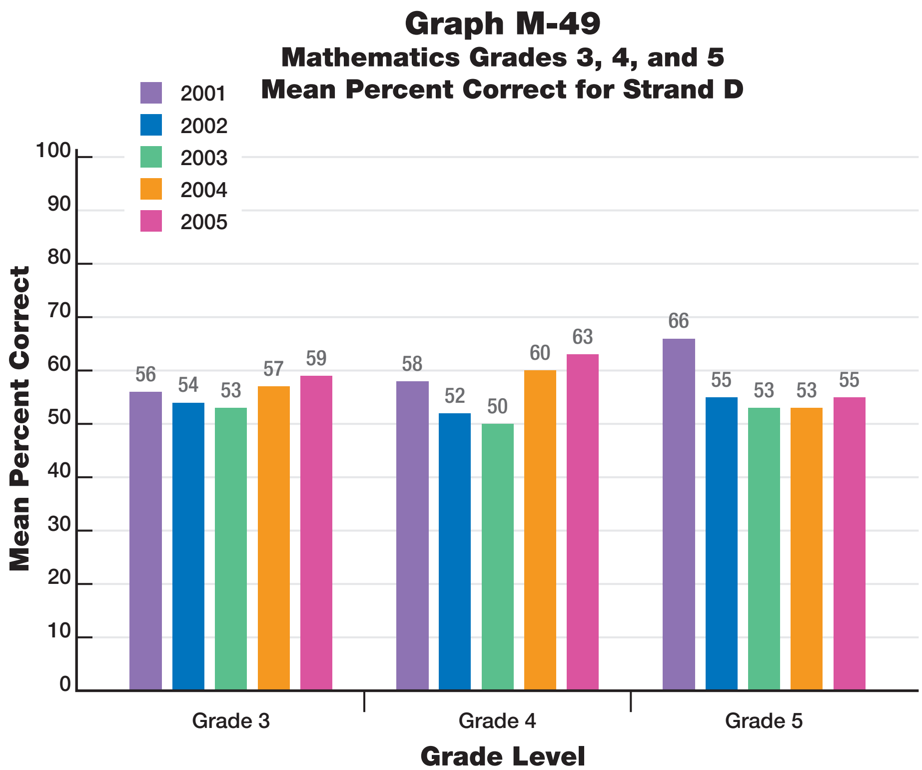
The task force recommends including the concepts of perpendicularity and parallelism in the algebra curriculum. The concept of slope and its meaning in context of the problem should be emphasized. Students should practice problems that require the use of the Pythagorean theorem when the hypotenuse of a right triangle on a coordinate grid is provided and the length of one of the legs must be calculated.

Strand D–*Algebraic Thinking*

Grades 3–5

Strand D Results for Grades 3–5

The following graph provides results on Strand D (*Algebraic Thinking*) for Grades 3–5.



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Algebraic Thinking* standards and benchmarks for Grades 3–5. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Strand D

Standard D1: The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.
Benchmark MA.D.1.2.1: The student describes a wide variety of patterns and relationships through models, such as manipulatives, tables, graphs, rules using algebraic symbols. (Also assesses D.1.2.2)
Benchmark MA.D.1.2.2: The student generalizes a pattern, relation, or function to explain how a change in one quantity results in a change in another. (Also assesses D.1.2.1) (Grade 3–4: Not assessed)

Grade 3: Observations for Standard D1–*Algebraic Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify the missing element in linear patterns with equal intervals or extend the pattern to the next step;
- identify the missing element in both increasing and decreasing linear patterns with equal intervals; and
- identify the missing element in a simple graphic pattern or extend a graphic pattern to the next step.

Students who are **unsuccessful** have the greatest difficulty with

- finding a relationship between two sets of objects when the relationship is not shown in a table format (e.g., using a balance scale to show that 3 white cubes balancing 1 black cube is the same as 6 white cubes balancing 2 black cubes).

Grade 4: Observations for Standard D1–*Algebraic Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify the missing element in linear patterns with equal intervals or extend the pattern to the next step;
- identify the missing element in both increasing and decreasing consecutive linear patterns; and
- identify the missing element in a simple graphic pattern or extend a graphic pattern to the next step.

Students who are **unsuccessful** have the greatest difficulty with

- finding an element that extends two or three steps beyond what is given in the pattern.

Grade 5: Observations for Standard D1–*Algebraic Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- extend graphic patterns beyond the next element.

Students who are **unsuccessful** have the greatest difficulty with

- extending numerical patterns going beyond the next element in a pattern (see sample item on the following page);
- generalizing the relationship between the independent and dependent variables, especially when the pattern is not consecutive;
- applying the pattern, or relationship, to find missing elements in a pattern after identifying the relationship (e.g., even if students are able to find the relationship between the number of cups of milk to the number of eggs in a recipe for four servings, they have trouble extending that to six or eight servings).

The following is a sample of a short-response task and a top-score response.

THINK SOLVE EXPLAIN

Tracie is making soup. The recipe includes the table below.

RICE AMOUNTS

Number of Soup Servings	Number of Cups of Rice
3	$1\frac{1}{2}$
6	3
9	$4\frac{1}{2}$
12	6
15	

Part A Complete the pattern on the table to show the number of cups of rice Tracie should use for 15 servings of soup. Use the space below to show your work or explain how you determined your answer.

An explanation or work similar to the following:

The number of cups of rice is half of the number of soup servings

$6 \div 2 = 3$ and $12 \div 2 = 6$

so $15 \div 2 = 7\frac{1}{2}$

Cups of Rice $7\frac{1}{2}$

Part B On the lines below, explain how to find the number of cups of rice for any number of servings of soup.

An explanation similar to the following:

To find the number of cups of rice, divide the number of soup servings by 2.

Most recent student results

14% earned 2 points
61% earned 1 point
26% earned 0 points

Grades 3–5: Implications for Instruction for Standard D1–*Algebraic Relationships*

The task force recommends that, when working with patterns and relationships in a two-column table format, teachers should emphasize generalizing the relationship between two variables, rather than only finding a pattern in a consecutive set of numbers in one of the columns. Students should practice applying that relationship to find a missing number in a set that does not have a linear pattern with equal intervals. Students should also practice finding a number that is extended beyond the next step in a linear pattern. This leads to the more formal introduction of functions in later grades. As is the case in many instances, the task force stresses the importance of using manipulatives to work through problems involving patterns and relationships. The task force recommended continued emphasis on reading skills in the mathematics curriculum to improve student performance on items set in a real-world context.

Standard D2: The student uses expressions, equations, inequalities, graphs, and formulas to represent and interpret situations.
Benchmark MA.D.2.2.1: The student represents a given simple problem situation using diagrams, models, and symbolic expressions translated from verbal phrases, or verbal phrases translated from symbolic expressions, etc. (Also assesses D.2.2.2)
Benchmark MA.D.2.2.2: The student uses informal methods, such as physical models and graphs, to solve real-world problems involving equations and inequalities. (Also assesses D.2.2.1)

Grade 3: Observations for Standard D2–*Solving and Interpretation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- translate word problems involving addition into expressions (see sample item on the following page); and
- solve equations involving addition.

Students who are **unsuccessful** have the greatest difficulty with

- choosing the correct operation to solve a word problem when key words are not explicitly indicated;
- translating word problems into equations or expressions that involve multiplication or division;
- recognizing that equations can be written in several different formats to solve for the same variable (e.g., $14 + r = 29$ and $29 - 14 = r$ can both be used to find the value of r);
- understanding the meaning of equivalence and equations;
- balancing equations that include geometric shapes; and
- understanding the meaning of inequalities and how to translate word problems into inequalities.

Sentis had 4 stickers. She bought more at the store. Now she has 20 stickers.

Which number sentence can be used to represent the number of stickers she bought at the store?

☞ A. 4 + □ = 20
B. 20 + 4 = □
C. 4 × □ = 20
D. □ − 4 = 20

☞ Correct answer

Most recent student results

58% chose option A
17% chose option B
21% chose option C
4% chose option D

Grade 4: Observations for Standard D2–*Solving and Interpretation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- understand that the concept of *twice* means to multiply by 2;
- use letter variables in equations, rather than geometric symbols; and
- translate simple one-step word problems into expressions or equations.

Students who are **unsuccessful** have the greatest difficulty with

- distinguishing between the meanings of multiplication and division;
- solving multiple-step word problems;
- balancing equations that include geometric shapes instead of numbers; and
- demonstrating an understanding of equivalence through appropriate use of the equal sign.

Ellen saw an ad for dinner specials at her favorite restaurant. The ad said all dinners were less than $8.

Which of the following **best** represents the price of one dinner (d) at this restaurant?

☞ A. $d < 8$
B. $d > 8$
C. $d \leq 8$
D. $d = 8$

☞ Correct answer

Most recent student results

47% chose option A
27% chose option B
8% chose option C
18% chose option D

Grade 5: Observations for Standard D2–*Solving and Interpretation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- choose the correct operation needed to solve an equation; and
- identify which inequality best represents data shown in a table.

Students who are **unsuccessful** have the greatest difficulty with

- balancing equations that include geometric shapes rather than numbers;
- understanding the meaning of equivalence and balancing an equation;
- translating word problems to inequalities (understanding the meaning of the ≥ and the ≤ signs), especially when the word problems do not include key words, such as *greater than or equal to*, or *less than or equal to* (see sample item above); and
- solving multiple-step word problems involving algebraic thinking.

Strand D

Grades 3–5: Implications for Instruction for Standard D2–*Solving and Interpretation*

For Grades 3–5, the task force recommends that teachers stress critical reading skills in mathematics. Students should review and interpret the entire problem before attempting to translate a word problem into an equation, expression, or inequality, or attempting to solve a problem. Students should understand that word problems do not always contain expected key words and there are often several different ways to represent the symbolic translation of a problem, as well as several different ways to solve the same problem. Students should check for the reasonableness of their solutions at all times.

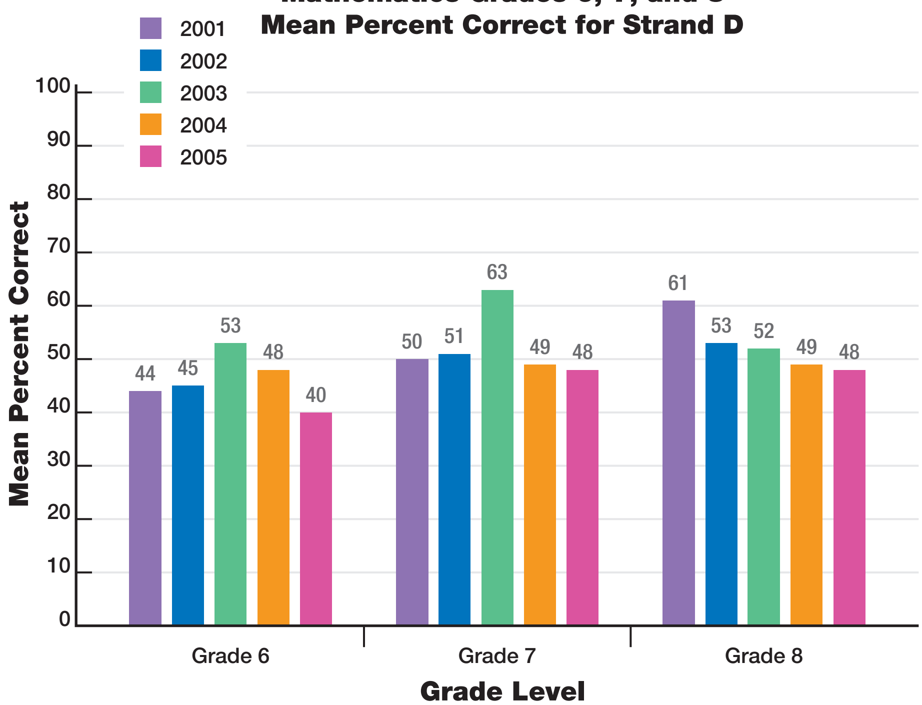
In addition, students in Grades 4 and 5 should have more instruction in the meaning of the symbols ≥ and ≤, and the terminology associated with the symbols.

Grades 6–8

Strand D Results for Grades 6–8

The following graph illustrates the performance of students in Grades 6–8 on the same strand (*Algebraic Thinking*).

Graph M-50
Mathematics Grades 6, 7, and 8
Mean Percent Correct for Strand D



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Algebraic Thinking* standards and benchmarks for Grades 6–8. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Strand D

Standard D1: The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.
Benchmark MA.D.1.3.1: The student describes a wide variety of patterns, relationships, and functions through models, such as manipulatives, tables, graphs, expressions, equations, and inequalities. (Also assesses A.5.3.1)
Benchmark MA.D.1.3.2: The student creates and interprets tables, graphs, equations, and verbal descriptions to explain cause-and-effect relationships. (Also assesses A.5.3.1)

Grade 6: Observations for Standard D1–*Algebraic Relationships*

Analysis of student performance data reveals the following:

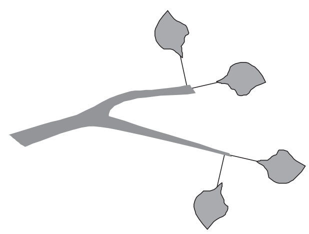
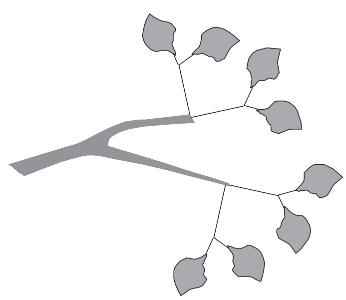
Students who are **successful** are able to

- interpret graphs; and
- extend a graphic pattern (see sample item on the following page).

Students who are **unsuccessful** have the greatest difficulty with

- completing multiple-step problems, especially those involving patterns with more than one operation to transform one term into the next;
- extending a numerical pattern;
- identifying the correct graph or table from a written description;
- describing an equivalent expression;
- understanding that a coefficient with a variable implies multiplication (e.g., $3x$ means "3 times x");
- translating from a written description or table into an equation; and
- evaluating an expression with exponents.

In four cycles of growth, a tree limb shows the following branching pattern.

Cycles		Growth Tips
1		1
2		2
3		4
4		8

If the tree limbs continue to branch at their current growth rate, how many growth tips will there be after two additional growth cycles?

A. 12
B. 16
C. 24
☞ D. 32

☞ Correct answer

Most recent student results

15% chose option A
26% chose option B
11% chose option C
49% chose option D

Strand D

Grade 7: Observations for Standard D1–*Algebraic Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- select the correct equation that describes the relationship between values in a table;
- select the correct expression that represents a real-world problem (see sample item below);
- interpret a table of values and identify the correct equation;
- interpret simple line graphs;
- evaluate an expression containing variables, given the value(s) of the variable(s); and
- complete a table or extend a pattern in a table.

Students who are **unsuccessful** have the greatest difficulty with

- continuing a graphic pattern and predicting several iterations;
- interpreting the data in a table to identify the correct graph in both mathematical and real-world contexts;
- translating written descriptions into the correct equation; and
- interpreting inequalities from a graph.

Sharon wants to join a mail order music club that offers CDs for $11.95 each. Each order must be accompanied by $6.95 for shipping. If x equals the number of CDs ordered, which expression below describes how much Sharon will owe?

A. $11.95x$
B. $18.90x$
C. $11.95(x + 6.95)$
☞ D. $11.95x + 6.95$

☞ Correct answer

Most recent student results

4% chose option A
22% chose option B
20% chose option C
54% chose option D

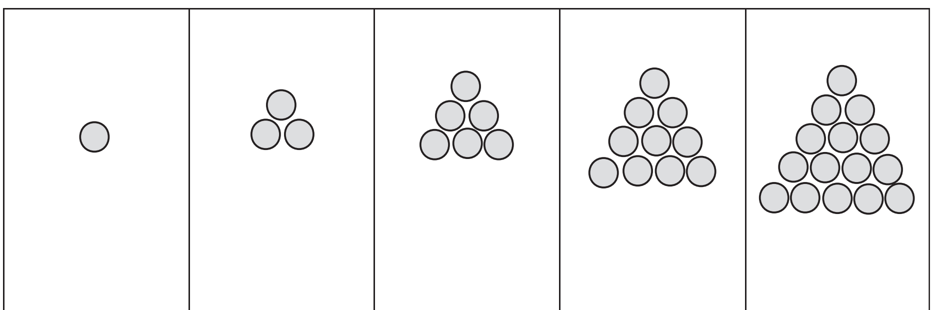
Grade 8: Observations for Standard D1–*Algebraic Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- interpret a table or recognize a pattern in a table to make accurate predictions;
- complete a table of values when given a graph;
- continue a graphic pattern (see sample item below);
- identify the correct expression for a word problem with one variable; and
- evaluate an expression containing variables when given the value(s) of the variable(s).

Students who are **unsuccessful** have the greatest difficulty with
- selecting the correct expression when given a picture and table (e.g., using multiple representations to write an expression);
- identifying correct expressions with two variables given a verbal description;
- extending nonlinear, irregular graphic patterns;
- writing an inequality to describe a given graph on a number line; and
- building from a table or pattern to predict subsequent values.

Triangular numbers, often used in making artistic designs, are shown below.



Value	1	3	6	10	15
Term	1st	2nd	3rd	4th	5th

If the pattern continues, what is the value of the 15th term in this sequence?

Correct answer: 120

Most recent student results

49% of Grade 8 students answered this problem correctly.

Grades 6–8: Implications for Instruction for Standard D1—*Algebraic Relationships*

For Grade 6, students need a variety of experiences translating written problems into algebraic representations. Students also need a variety of experiences describing, creating, and interpreting charts and tables.

For Grade 7, students need practice extending patterns beyond two or three iterations and experience describing patterns and inequalities.

For Grade 8, students need practice with patterns and sequences other than arithmetic and geometric. Students also need a variety of experiences selecting and identifying expressions.

For Grades 6–8, the task force recommends that students use handheld or computer technology to demonstrate multiple representations (table, algebraic expression or equation, graph) for a given problem.

Standard D2: The student uses expressions, equations, inequalities, graphs, and formulas to represent and interpret situations.
Benchmark MA.D.2.3.1: The student represents and solves real-world problems graphically, with algebraic expressions, equations, and inequalities. (Also assesses A.1.3.3)
Benchmark MA.D.2.3.2: The student uses algebraic problem-solving strategies to solve real-world problems involving linear equations and inequalities.

Grade 6: Observations for Standard D2—*Solving and Interpretation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- solve a simple equation; and
- solve real-world problems involving algebraic expressions.

Students who are **unsuccessful** have the greatest difficulty with
- applying grouping symbols;
- identifying expressions with one or more variables and multiple operations;
- translating and solving equations with variables on the left-hand side of the equation;
- identifying an inequality that represents a problem situation; and
- solving a problem with multiple variables.

Grade 7: Observations for Standard D2–*Solving and Interpretation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- solve an equation for given value(s) of the variable(s).

Students who are **unsuccessful** have the greatest difficulty with

- identifying an equation that corresponds to information given in a table;
- solving distance, rate, and time problems when the information is not explicitly stated;
- understanding the appropriate use of iterations involving subtraction (e.g., $n - 1$, $n - 2$); and
- translating a written description into an expression, equation, or inequality.

Grade 8: Observations for Standard D2–*Solving and Interpretation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify an equation that corresponds to information given in a table;
- solve an equation for given value(s) of the variable(s);
- solve an equation that does not include fractions; and
- identify the equation that best models a scatter plot.

Students who are **unsuccessful** have the greatest difficulty with

- translating a written description into an expression, equation, or inequality;
- graphing inequalities on a number line involving open intervals versus closed intervals;
- fractional coefficients in an equation (e.g., $\frac{1}{3}x$);
- order of operations;
- creating an equation to solve percent increase or decrease problems; and
- ignoring extraneous information when identifying an equation.

Grades 6–8: Implications for Instruction for Standard D2–*Solving and Interpretation*

The task force recommends that teachers encourage students to use critical reading strategies, such as marking the text where appropriate, so they can answer the question that is being asked. Teachers should let students share their strategies with peers for problem set-up. Students should develop the habit of checking their answers and practice other success strategies. Students should also have frequent opportunities to practice reading to identify important information in a problem-solving context.

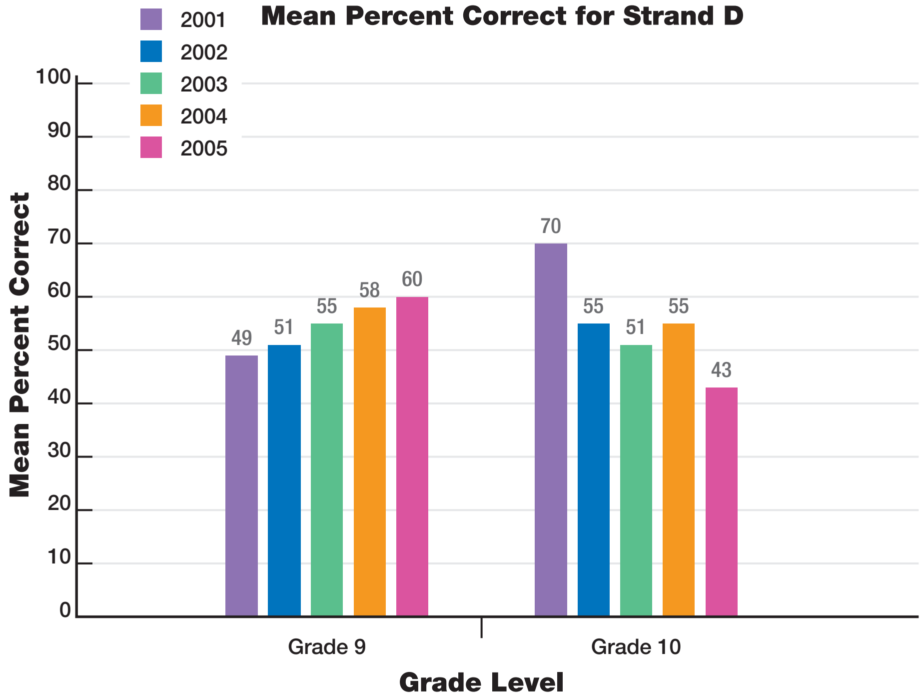
 |

Grades 9–10

Strand D Results for Grades 9–10

The following graph illustrates the performance of students in Grades 9–10 on the same strand (*Algebraic Thinking*).

Graph M-51
Mathematics Grades 9 and 10
Mean Percent Correct for Strand D



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Algebraic Thinking* standards and benchmarks for Grades 9–10. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Standard D1: The student describes, analyzes, and generalizes a wide variety of patterns, relations, and functions.
Benchmark MA.D.1.4.1: The student describes, analyzes, and generalizes relationships, patterns, and functions using words, symbols, variables, tables, and graphs.
Benchmark MA.D.1.4.2: The student determines the impact when changing parameters of given functions.

Strand D

Grade 9: Observations for Standard D1–*Algebraic Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- identify the equation of a linear function from information given in chart form (see sample item below);
- recognize the impact of changing parameters in linear functions; and
- complete graphic patterns.

Students who are **unsuccessful** have the greatest difficulty with

- identifying equations of lines from line graphs that represent real-world relationships (e.g., non-routine scales); and
- reading and understanding all details of a question.

A movie theater offers tickets for sale on the Internet at a discounted price. Each individual ticket costs the same amount, and there is a fixed processing fee for each Internet purchase of one or more tickets. The table below shows the total cost for purchasing different numbers of tickets from this movie theater on the Internet.

MOVIE THEATER TICKET PURCHASES ON THE INTERNET

Number of Tickets Purchased (t)	Total Cost (C)
1	\$ 6.50
2	\$11.75
3	\$17.00
4	\$22.25

Using the data in the table, which equation can be used to find the total cost, C, to purchase t tickets from this movie theater on the Internet?

A. $C = 6.50t$
B. $C = 5.56t$
C. $C = 5.50t + 1.00$
☞ D. $C = 5.25t + 1.25$

☞ Correct answer

Most recent student results

31% chose option A
6% chose option B
13% chose option C
50% chose option D

Strand D

Grade 10: Observations for Standard D1–*Algebraic Relationships*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- calculate the change in a parameter, given a specific equation and specific information (see sample item below).

Students who are **unsuccessful** have the greatest difficulty with

- recognizing relationships in a geometric sequence.

Jennifer bought a new energy-saving fluorescent light bulb to replace the standard light bulb for her backyard floodlight. The table below shows the energy usage for each type of light bulb.

LIGHT BULB ENERGY USAGE

Type of Light bulb	Energy Usage (in watts)
Fluorescent light bulb	65
Standard light bulb	300

Jennifer wants to use the following equation to find the cost to operate each type of light bulb.

$$C = 0.001\ (whr)$$

where:

C = the total cost, in dollars
h = the number of hours used
w = the number of watts
r = the charge per kilowatt-hour, in dollars

The electric company currently charges $0.06275 per kilowatt-hour. Which of the following is closest to the amount of money Jennifer should expect to save if she uses the fluorescent light bulb instead of the standard light bulb **9 hours each day for 30 days**?

A. $0.13
B. $1.10
☞ C. $3.98
D. $5.08

☞ Correct answer

Most recent student results

10% chose option A
27% chose option B
46% chose option C
13% chose option D

Grades 9–10: Implications for Instruction for Standard D1–*Algebraic Relationships*

The task force recommends that students acquire more practice with rules that apply to sequences of numbers and nonlinear number patterns. More attention should be given to vocabulary (e.g., squaring, doubling, finding half of), and students should be exposed to more nonroutine problem-solving situations. Students perform well when finding what comes next in a pattern, but still need more work generalizing the pattern, rule, and function.

Standard D2: The student uses expressions, equations, inequalities, graphs, and formulas to represent and interpret situations.
Benchmark MA.D.2.4.1: The student represents real-world problem situations using finite graphs, matrices, sequences, series, and recursive relations. (Assessed with C.3.4.2 and D.2.4.2)
Benchmark MA.D.2.4.2: The student uses systems of equations and inequalities to solve real-world problems graphically, algebraically, and with matrices. (Also assesses D.2.3.1, D.2.3.2, and D.2.4.1)

Grade 9: Observations for Standard D2–*Solving and Interpretation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- solve real-world problems in one variable involving one or two steps.

Students who are **unsuccessful** have the greatest difficulty with

- understanding the conceptual meaning of inequalities; and
- solving real-world problems in one variable involving more than two steps.

Grade 10: Observations for Standard D2–*Solving and Interpretation*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- solve real-world problems in one variable involving more than two steps;
- choose a system of equations or inequalities to fit a given situation; and
- solve a given system of equations by direct substitution.

Students who are **unsuccessful** have the greatest difficulty with

- creating a system of equations or inequalities and solving it; and
- solving systems of equations with non-integral coefficients.

Grades 9–10: Implications for Instruction for Standard D2–*Solving and Interpretation*

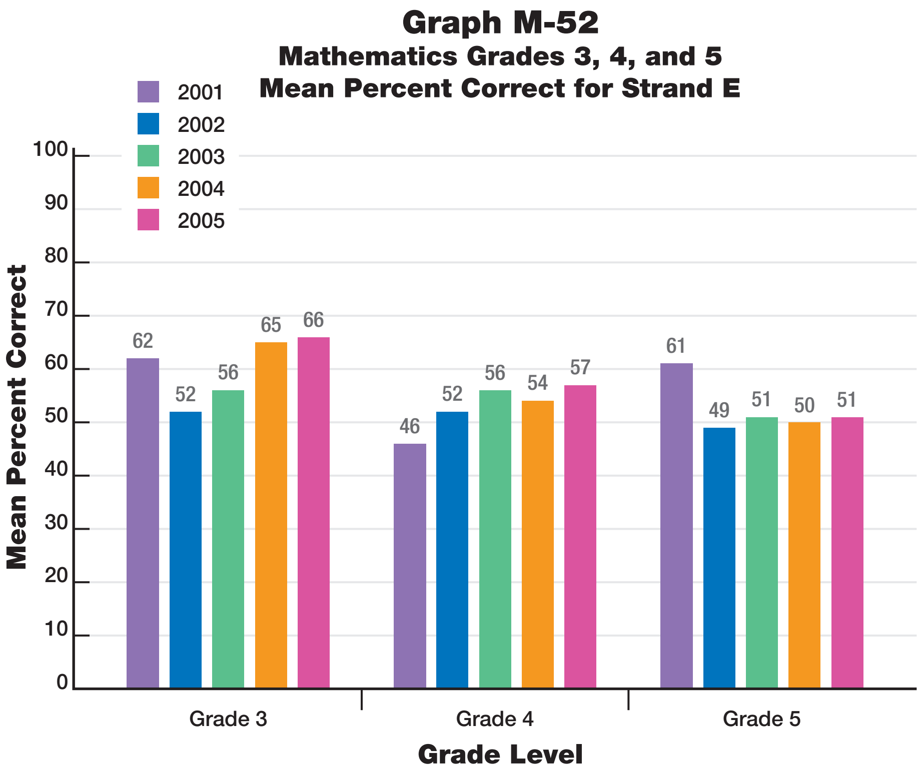
The task force recommends that students be exposed to opportunities to solve a problem in multiple ways and have additional practice translating words to symbols and words to equations, expressions, and inequalities.

Strand E–*Data Analysis and Probability*

Grades 3–5

Strand E Results for Grades 3–5

The following graph provides results on Strand E *(Data Analysis and Probability)* for Grades 3–5.



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Data Analysis and Probability* standards and benchmarks for Grades 3–5. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Standard E1: The student understands and uses the tools of data analysis for managing information.
Benchmark MA.E.1.2.1: The student solves problems by generating, collecting, organizing, displaying, and analyzing data using histograms, bar graphs, circle graphs, line graphs, pictographs, and charts. (Also assesses E.1.2.3)
Benchmark MA.E.1.2.2: The student determines range, mean, median, and mode from sets of data. (Also assesses E.1.2.3)
Benchmark MA.E.1.2.3: The student analyzes real-world data to recognize patterns and relationships of the measures of central tendency using tables, charts, histograms, bar graphs, line graphs, pictographs, and circle graphs generated by appropriate technology, including calculators and computers. (Assessed with E.1.2.1 and E.1.2.2)

Grade 3: Observations for Standard E1–*Data Analysis Tools*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify and compare information in bar graphs;
- identify and compare information in pictographs; and
- identify the mode of a set of numbers.

Students who are **unsuccessful** have the greatest difficulty with
- determining the median when numbers are not ordered;
- determining the range when numbers are not ordered (see sample item on the following page); and
- interpreting pictographs with only partial representation, particularly with larger numbers greater than one thousand.

Calli weighed 7 different kinds of rocks. She labeled the rocks with letters and made the table below to show the weight of each rock.

ROCK WEIGHTS

Rock	A	B	C	D	E	F	G
Weight (in ounces)	6	8	5	8	8	9	7

What is the **range** of the rock weights?

A. 3 ounces
☞ B. 4 ounces
C. 8 ounces
D. 9 ounces

☞ Correct answer

Most recent student results

10% chose option A
39% chose option B
33% chose option C
18% chose option D

Grade 4: Observations for Standard E1–*Data Analysis Tools*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- make simple interpretations of information in bar graphs; and
- identify the mode and median of a set of numbers.

Students who are **unsuccessful** have the greatest difficulty with
- analyzing bar graphs when a two-step calculation is required;
- determining appropriate intervals of the scale on a bar graph;
- determining the median when numbers are not ordered; and
- determining the range and mean of a set of numbers.

  |

Grade 5: Observations for Standard E1—*Data Analysis Tools*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- calculate mean in simple situations (see sample item below);
- make simple interpretations of information in bar graphs and line graphs; and
- find the missing percents in a circle graph.

Students who are **unsuccessful** have the greatest difficulty with

- calculating the median of a data set with an even number of data points; and
- converting data into percents and then transferring the percents to circle graphs.

Rocco feeds turtles at the pond in his backyard. During a 7-day period, Rocco counted the number of turtles he saw.

TURTLES SEEN

Day	Number
Monday	9
Tuesday	14
Wednesday	6
Thursday	5
Friday	10
Saturday	5
Sunday	7

What is the **mean** number of turtles seen during this week?

Correct answer: 8

Most recent student results

51% of Grade 5 students answered this problem correctly.

Strand E

Grades 3–5: Implications for Instruction for Standard E1–*Data Analysis Tools*

For Grade 3, the task force recommends that, when teaching pictographs, students should practice with partial representations of the units in the legend. Practice should start with smaller numbers and increase to larger numbers (e.g., single to double to triple digits) within content limits.

For Grade 4, students should practice using multiple-step problems with bar graphs where they must manipulate the data. Students should have experience calculating and applying the concepts of range and median when numbers are not in order. Teachers should provide opportunities for students to work problems involving multiple formats, graphs, and tables.

For Grade 5, teachers should stress number sense and percents and emphasize reasonableness of answers. Students should apply range and mean concepts with multiple-step directions. Students should practice the use of all types of graphs, tables, and charts. Students should practice finding the median of a set of data with an even number of data points.

Standard E2: The student identifies patterns and makes predictions from an orderly display of data using concepts of probability and statistics.
Benchmark MA.E.2.2.1: The student uses models, such as tree diagrams, to display possible outcomes and to predict events.

Grade 3: Observations for Standard E2–*Patterns and Predictions*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- identify the least likely and most likely probabilities; and
- calculate simple probabilities.

Students who are **unsuccessful** have the greatest difficulty with
- finding outcomes and combinations, such as listing a combination of two from a set of four (see sample item on the following page).

Grade 4: Observations for Standard E2–*Patterns and Predictions*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- use pictures (not tables) when finding combinations;
- identify the least likely and most likely probabilities; and
- calculate simple probabilities.

Students who are **unsuccessful** have the greatest difficulty with

- identifying combinations from a table; and
- identifying probabilities with spinners divided into unequal parts.

A group of fourth graders is designing a flag. They may choose 2 of these colors.

RED **YELLOW** **BLACK** **GREEN**

How many different 2-color combinations can be made?

A. 3
B. 4
☞ C. 6
D. 8

☞ Correct answer

Most recent student results

9% chose option A
20% chose option B
41% chose option C
31% chose option D

Strand E

Grade 5: Observations for Standard E2–*Patterns and Predictions*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- calculate probability that does not require simplification of fractions; and
- identify the least likely and most likely probabilities.

Students who are **unsuccessful** have the greatest difficulty with
- probability problems with the word *or* (e.g., finding the possibility that a red or a green object is chosen from a group);
- determining probabilities that require simplifying fractions; and
- creating tree diagrams to find the number of combinations.

Grades 3–5: Implications for Instruction for Standard E2–*Patterns and Predictions*

For Grades 3–5, the task force recommends emphasis on using tables and other methods to find outcomes, as opposed to just counting them. Students should develop their vocabulary of outcomes and combinations to better understand the concepts.

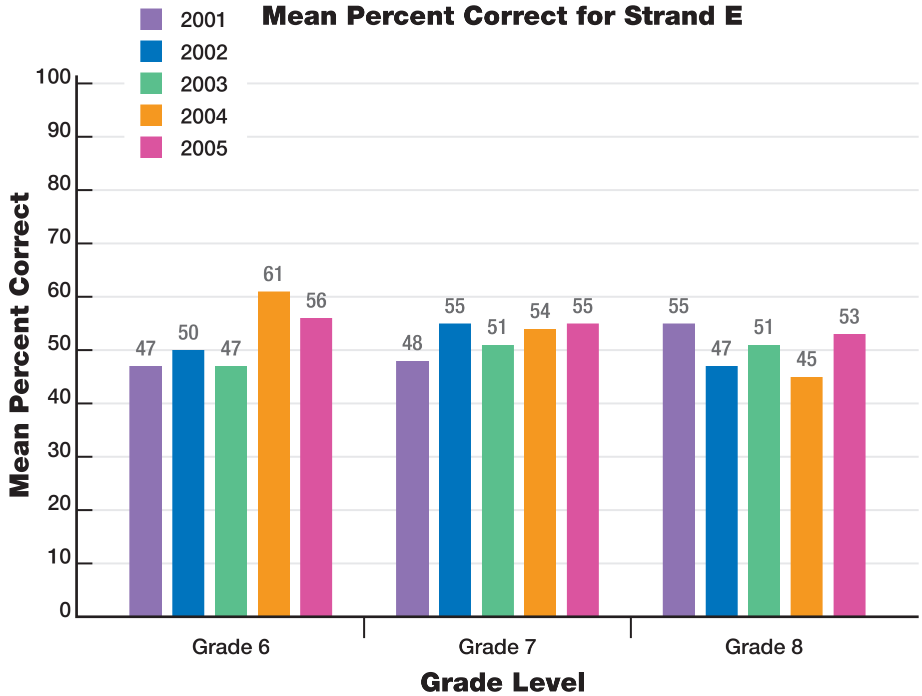
For Grade 5, students should practice calculating probability with *and* and *or* statements. Teachers should emphasize practice with spinners having unequal parts. Spinners having unequal parts should be compared to spinners with equal parts, so that students will have a better understanding of the meanings of *most likely, least likely,* and *equal* outcomes. Students should practice working with tree diagrams and charts to represent combinations as well as making lists.

Grades 6–8

Strand E Results for Grades 6–8

The following graph illustrates the performance of students in Grades 6–8 on the same strand (*Data Analysis and Probability*).

Graph M-53
Mathematics Grades 6, 7, and 8
Mean Percent Correct for Strand E



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Data Analysis and Probability* standards and benchmarks for Grades 6–8. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Strand E

Standard E1: The student understands and uses the tools of data analysis for managing information.
Benchmark MA.E.1.3.1: The student collects, organizes, and displays data in a variety of forms, including tables, line graphs, charts, bar graphs, to determine how different ways of presenting data can lead to different interpretations. (Also assesses E.1.3.3)
Benchmark MA.E.1.3.2: The student understands and applies the concepts of range and central tendency (mean, median, and mode). (Also assesses E.1.3.3)
Benchmark MA.E.1.3.3: The student analyzes real-world data by applying appropriate formulas for measures of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers. (Assessed with E.1.3.1 and E.1.3.2)

Grade 6: Observations for Standard E1–*Data Analysis Tools*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

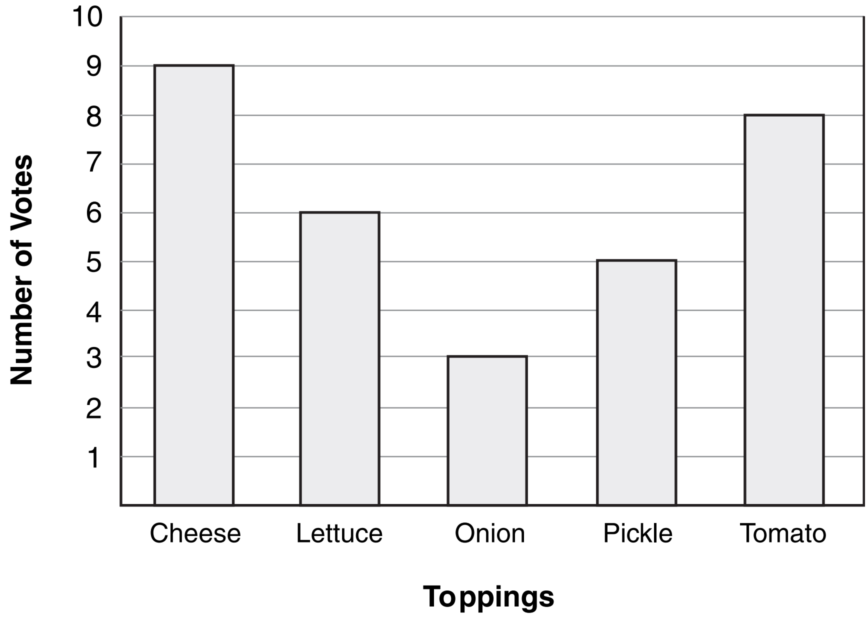
- read and interpret bar graphs, scatter plots, pictographs, and stem-and-leaf plots (see sample item on the following page); and
- find the mode.

Students who are **unsuccessful** have the greatest difficulty with

- multiple-step problems with graphs or charts, especially circle graphs;
- interpreting Venn diagrams; and
- calculating the mean on open-ended, gridded-response items especially when decimal values are involved.

Each student in Mr. Villarreal's class voted for one favorite hamburger topping. Mr. Villarreal constructed the following bar graph based on their votes.

FAVORITE HAMBURGER TOPPINGS



How many **more** students preferred cheese **or** lettuce rather than onions **or** pickles?

Correct answer: 7

Most recent student results

54% of Grade 6 students answered this problem correctly.

Grade 7: Observations for Standard E1–*Data Analysis Tools*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- read and interpret bar graphs, scatter plots, pictographs, and stem-and-leaf plots; and
- find the mean (see sample item on the following page).

Students who are **unsuccessful** have the greatest difficulty with
- multiple-step problems with graphs and charts, especially circle graphs;
- interpreting Venn diagrams; and
- calculating the median given an even number of data points.

Scientists have identified a number of dinosaurs that lived during the Jurassic Period. The chart shows the scientific names of some of the dinosaurs believed to have lived in North America during this time and their approximate maximum length in feet.

DINOSAURS OF THE JURASSIC PERIOD IN NORTH AMERICA

Dinosaur	Maximum Length (feet)
Apatosaurus	70
Allosaurus	52
Barosaurus	80
Camarasaurus	65
Camptosaurus	16
Diplodocus	91
Dryosaurus	13
Ornitholestes	6
Stegosaurus	30

Which maximum length in the table is closest to the **mean**?

Correct answer: 52

Most recent student results

58% of Grade 7 students answered this problem correctly.

Strand E

Grade 8: Observations for Standard E1–*Data Analysis Tools*

Analysis of student performance data reveals the following:

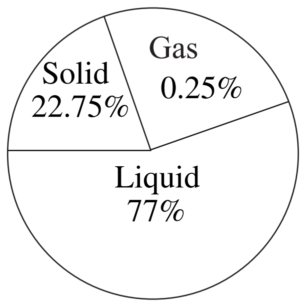
Students who are **successful** are able to

- read and interpret bar graphs, scatter plots, pictographs, circle graphs, line graphs, and stem-and-leaf plots (see sample item on the following page);
- determine the most appropriate representation to display data; and
- find the median of an odd number of data points.

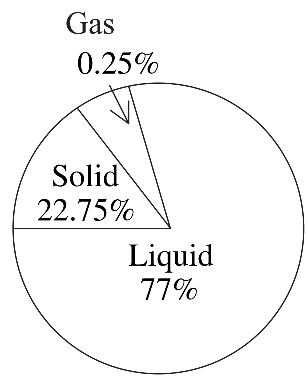
Students who are **unsuccessful** have the greatest difficulty with

- multiple-step problems with graphs or charts, especially circle graphs;
- interpreting Venn diagrams; and
- finding the mean in a multiple-step problem or finding a value that will yield a given mean.

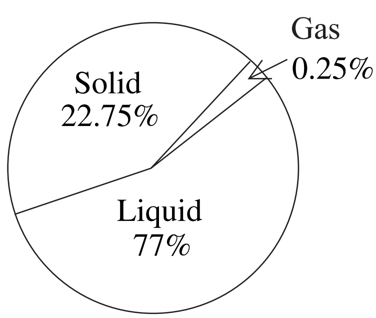
The majority of Earth's surface consists of water. Water on Earth exists in three different forms: liquid (77%), solid (22.75%), and gas (0.25%). Which of the following circle graphs most closely illustrates these three water forms?



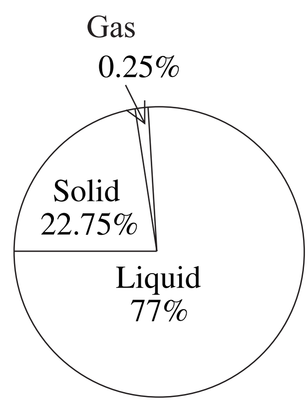
A.



C.



B.



☞ D.

☞ Correct answer

Most recent student results

14% chose option A
11% chose option B
25% chose option C
50% chose option D

Strand E

Grades 6–8: Implications for Instruction for Standard E1–*Data Analysis Tools*

For Grade 6, teachers should stress the use of vocabulary necessary to interpret line graphs (e.g., constant, rate of increase, rate of decrease).

For Grade 7, students should experience calculating measures of central tendency, especially finding a median with an even number of data points.

For Grades 6–8, the task force recommends that students acquire additional experience with problems that involve, but are not limited to, probability. Students also need experience in making inferences and drawing conclusions after performing calculations based on data displays.

Standard E2: The student identifies patterns and makes predictions from an orderly display of data using concepts of probability and statistics.
Benchmark MA.E.2.3.1: The student compares experimental results with mathematical expectations of probabilities.

Grade 6: Observations for Standard E2–*Patterns and Predictions*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- calculate simple probabilities.

Students who are **unsuccessful** have the greatest difficulty with
- understanding the concept of combinations and applying them in real-world situations.

Grade 7: Observations for Standard E2–*Patterns and Predictions*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- calculate probabilities given two or more conditions; and
- calculate combinations given two or more conditions.

Students who are **unsuccessful** have the greatest difficulty with
- complex probabilities;
- complex combinations;
- determining odds; and
- finding probability when information is presented in a table and the total number of outcomes is not given.

Grade 8: Observations for Standard E2–*Patterns and Predictions*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- calculate simple probabilities.

Students who are **unsuccessful** have the greatest difficulty with

- finding probability when information is presented in a table and the total number of outcomes is not given;
- completing problems with the wording *not*; and
- determining odds for or against an event or an outcome.

Grades 6–8: Implications for Instruction for Standard E2–*Patterns and Predictions*

For Grades 7–8, students should practice finding probability with information presented in tables (when the total number of outcomes is not given). Students must understand the difference between probability and odds and appropriately apply each concept.

For Grades 6–8, the task force recommends that students acquire additional experience with problems that involve probability. Students also need experience making inferences and drawing conclusions after performing calculations based on data displays. Teachers include problems with *or* statements when teaching probability.

Standard E3: The student uses statistical methods to make inferences and valid arguments about real-world situations.
Benchmark MA.E.3.3.1: The student formulates hypotheses, designs experiments, collects and interprets data, and evaluates hypotheses by making inferences and drawing conclusions based on statistics (range, mean, median, and mode) and tables, graphs, and charts. (Also assesses E.3.3.2)
Benchmark MA.E.3.3.2: The student identifies the common uses and misuses of probability and statistical analysis in the everyday world. (Assessed with E.3.3.1)

Grade 6: Observations for Standard E3–*Statistical Methods*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- understand the relative size of information presented in a circle graph.

Students who are **unsuccessful** have the greatest difficulty with

- understanding the effects of an outlier on statistical descriptions of a set of values;
- making appropriate generalizations based on sample size and type; and
- making inferences with double-line graphs.

Grade 7: Observations for Standard E3–*Statistical Methods*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- make inferences derived from data presented in a simple form such as a table.

Students who are **unsuccessful** have the greatest difficulty with

- statements using the wording *not*;
- understanding the effects of an outlier on statistical descriptions of a set of values;
- making appropriate generalizations based on sample size and type; and
- making inferences with double-line graphs.

Grade 8: Observations for Standard E3–*Statistical Methods*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- make inferences derived from uncomplicated data.

Students who are **unsuccessful** have the greatest difficulty with

- understanding the effects of an outlier;
- making appropriate generalizations based on sample size and type; and
- making inferences with double-line graphs.

Grades 6–8: Implications for Instruction for Standard E3–*Statistical Methods*

The task force recommends that students acquire additional experience with problems that involve probability. Students also need experience in making inferences and drawing conclusions after performing calculations based on data displays. Teachers from other subjects, such as science and social sciences, should give students the opportunity to read and interpret data from charts, tables, and graphs. Students should work with data presented on a variety of graphs with different scales and intervals. Teachers should look for opportunities to incorporate concepts taught earlier related to mean, median, mode, and range.

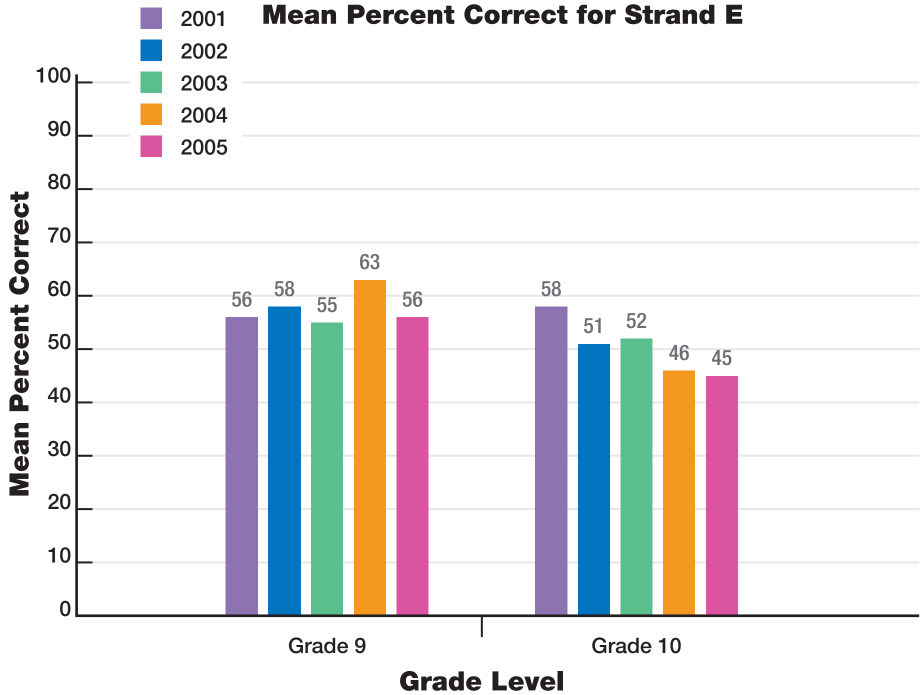
 |

Grades 9–10

Strand E Results for Grades 9–10

The following graph illustrates the performance of students in Grades 9–10 on the same strand (*Data Analysis and Probability*).

Graph M-54
Mathematics Grades 9 and 10
Mean Percent Correct for Strand E



Note: Caution must be used to interpret these graphs because the changes in performance over time may be attributed to changes in item difficulty. See pages 19–20 for a method of strand-level performance analysis for schools and districts.

The following charts show the *Data Analysis and Probability* standards and benchmarks for Grades 9–10. Each standard and its benchmarks are followed by observations, sample items, and implications for instruction.

Standard E1: The student understands and uses the tools of data analysis for managing information.
Benchmark MA.E.1.4.1: The student interprets data that has been collected, organized, and displayed in charts, tables, and plots. (Also assesses E.1.3.1 and E.1.4.3)
Benchmark MA.E.1.4.2: The student calculates measures of central tendency (mean, median, and mode) and dispersion (range, standard deviation, and variance) for complex sets of data and determines the most meaningful measure to describe the data. (Also assesses E.1.4.3)

Strand E

Grade 9: Observations for Standard E1–*Data Analysis*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- read data from bar graphs and tables;
- calculate central tendencies and range; and
- interpret Venn diagrams when asked for a basic interpretation (e.g., union of only 2 sets)

Students who are **unsuccessful** have the greatest difficulty with
- interpreting and applying measures of central tendencies and range;
- solving multiple-step problems involving measures of central tendency (see sample item below); and
- extracting information from or interpreting box-and-whisker plots.

The number of portable buildings produced by Chambers Manufacturing last week was 70 on Monday, 60 on Tuesday, 80 on Wednesday, and 50 on Thursday. After production on Friday, the mean number of buildings produced for the week was 67.

What is the **median** number of buildings produced last week by Chambers Manufacturing?

A. 67
B. 68
☞ C. 70
D. 75

☞ Correct answer

Most recent student results

37% chose option A
8% chose option B
43% chose option C
12% chose option D

Grade 10: Observations for Standard E1–*Data Analysis*

Analysis of student performance data reveals the following:

Students who are **successful** are able to
- calculate central tendency and range; and
- read and interpret data from various data displays.

Students who are **unsuccessful** have the greatest difficulty with
- multiple-step problems involving measures of central tendency and range; and
- extracting information from or interpreting box-and-whisker plots.

Grades 9–10: Implications for Instruction for Standard E1–*Data Analysis*

The task force recommends that teachers expose students to Venn diagram problems that use words similar to *at least* and *most*. Teachers should include practice on interpretation and application of results when calculating central tendencies and range (i.e., instruction must go beyond simply calculating).

Standard E2: The student identifies patterns and makes predictions from an orderly display of data using concepts of probability and statistics.
Benchmark MA.E.2.4.1: The student determines probabilities using counting procedures, tables, tree diagrams, and formulas for permutations and combinations. (Also assesses E.2.4.2)

Grade 9: Observations for Standard E2–*Patterns and Predictions*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- use the counting principle, tables, and tree diagrams to calculate the number of combinations and determine probability.

Students who are **unsuccessful** have the greatest difficulty with

- interpreting and computing the probability of compound, independent events.

Grade 10: Observations for Standard E2–*Patterns and Predictions*

Analysis of student performance data reveals the following:

Students who are **successful** are able to

- compute simple combinations (see sample item on the following page); and
- calculate simple probabilities involving single, independent events.

Students who are **unsuccessful** have the greatest difficulty with

- permutations;
- computing the probability of compound, dependent events; and
- understanding how permutations and/or combinations change depending on the situation.

Nine cards are numbered sequentially with the integers 1 through 9. The sum of the numbers on the first 2 cards selected at random is 13. If the cards are NOT replaced, what is the probability that the next card drawn will be an even number?

☞ A. $\frac{3}{7}$

B. $\frac{4}{7}$

C. $\frac{4}{9}$

D. $\frac{5}{9}$

☞ Correct answer

Most recent student results

54% chose option A
22% chose option B
18% chose option C
5% chose option D

Grades 9–10: Implications for Instruction for Standard E2–*Patterns and Predictions*

For Grade 9, the task force recommends that students gain an understanding of the meaning of probability by practicing different ways to approach finding probability; knowing which formula to apply is not enough. Students should understand why the formula works.

For Grade 10, students should understand how the concrete or hands-on examples relate to abstract or complex calculations. This concrete understanding should help students move beyond simply applying a formula. Students should practice with data sets that do not require using a formula in order to move beyond rote learning to a deeper understanding.

Standard E3: The student uses statistical methods to make inferences and valid arguments about real-world situations.
Benchmark MA.E.3.4.1: The student designs and performs real-world statistical experiments that involve more than one variable, then analyzes results and reports findings. (Also assesses E.3.3.1 and E.3.4.2)
Benchmark MA.E.3.4.2: The student explains the limitations of using statistical techniques and data in making inferences and valid arguments. (Assessed with E.3.4.1)

Grade 9: Observations for Standard E3–*Statistical Methods*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific recommendations about areas of student success.

Students who are **unsuccessful** have the greatest difficulty with

- evaluating data and applying it;
- understanding what is being asked in the test question;
- extrapolating and making predictions;
- interpolating and drawing conclusions;
- analyzing statistical results; and
- finding the graph or data display that best represents given data.

Grade 10: Observations for Standard E3–*Statistical Methods*

Analysis of student performance data reveals the following:

The items and student results reviewed by the task force did not warrant any specific recommendations about areas of student success.

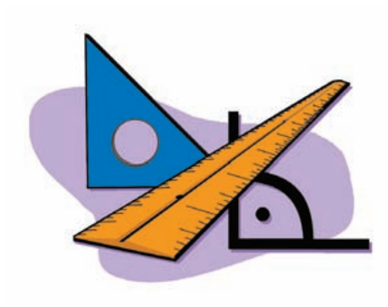
Students who are **unsuccessful** have the greatest difficulty with

- evaluating data and applying it;
- extrapolating and making predictions;
- interpolating and drawing conclusions;
- analyzing statistical results; and
- finding the graph or data display that best represents the given data.

Grades 9–10: Implications for Instruction for Standard E3–*Statistical Methods*

For Grade 9, the task force recommends instruction go beyond focus on a numerical answer (i.e., students should consider the information provided in the test question). Teachers should work with students to help them understand what is happening in the presented graph/diagram before attempting to do any calculations. Students should practice generalizing what the data mean and how they should be used.

For Grade 10, students need more opportunities to interpret graphs and manipulate data to make predictions and conclusions. Students should also practice identifying the correct type of graph to represent data.



Lessons Learned
CONCLUSION

FCAT Mathematics Lessons Learned Task Force Conclusions

There were task force recommendations that were common across the two *Lessons Learned* reports (2001 and 2007 versions). The following bulleted lists provide these common recommendations. Culling these results from the overall findings highlight opportunities for growth that have persisted over the eight years of data (1998 to 2005).

Strand A–*Number Sense, Concepts, and Operations*

Grades 3–5 (compare to Grade 5 in 2001 report)

- Students should get additional experience with word problems that require multiple steps.

Grades 6–8 (compare to Grade 8 in 2001 report)

- Students should practice problems that require persistence and thought.
- Students should practice multiple-step problems.
- Teachers should place more emphasis on problem solving.
- Students should get more practice creating an equation from a word problem.

Grades 9–10 (compare to Grade 10 in 2001 report)

- Students should practice interpreting and applying percentages needed in problem-solving situations.
- Students should practice using estimation in problem-solving situations.

Strand B–*Measurement*

Grades 3–5 (compare to Grade 5 in 2001 report)

- Students should practice finding length, weight, mass, capacity, and volume through hands-on activities using real-world objects.

Grades 6–8 (compare to Grade 8 in 2001 report)

- Students need experience that will help them develop visualization skills and allow them to understand the origin and application of formulas.
- Students should practice solving problems that involve multiple conversion steps and additional operations.

Grades 9–10 (compare to Grade 10 in 2001 report)

- Students should be given opportunities for hands-on activities to help build a foundation that can support understanding of the geometric concepts and link those concepts to the abstract.
- Students should continue to be encouraged to draw pictures to help them solve problems.

Strand C–*Geometry and Spatial Sense*

Grades 3–5 (compare to Grade 5 in 2001 report)

- Instruction on vocabulary and shapes should be provided at the same time.
- Students should use manipulatives to create shapes based on verbal or written descriptions.
- Teachers should create word walls for geometric terms.
- Teachers should use manipulatives to develop an understanding of reflections, translations, and rotations.
- Teachers should emphasize geometric terminology and provide hands-on practice using manipulatives.

Grades 6–8 (compare to Grade 8 in 2001 report)

- Students should be given opportunities to manipulate, draw, label, and construct geometric shapes.
- Students should understand the formal symbolism and vocabulary of geometry.
- Students should use manipulatives to experiment with the sum of the measures of polygons.

Grades 9–10 (compare to Grade 10 in 2001 report)

- Students need more practice expressing their thinking using correct mathematical terminology.
- Students should build on the informal reasoning learned in earlier grades to develop more formal reasoning.

Strand D–*Algebraic Thinking*

Grades 3–5 (compare to Grade 5 in 2001 report)

- There were no common recommendations for this standard and grade range.

Grades 6–8 (compare to Grade 8 in 2001 report)

- Students need a variety of experiences with translating written problems into algebraic representations.

Grades 9–10 (compare to Grade 10 in 2001 report)

- More attention needs to be given to vocabulary (e.g., *squaring, doubling, finding half of*), and students should be exposed to more non–routine, problem–solving situations.
- Students need additional practice translating words to symbols and words to equations, expressions, and inequalities.

Strand E–*Data Analysis and Probability*

Grades 3–5 (compare to Grade 5 in 2001 report)

- Students should practice making lists and working with tree diagrams and charts to represent combinations.

Grades 6–8 (compare to Grade 8 in 2001 report)

- Students need experience with making inferences and drawing conclusions after performing calculations based on data displays.
- Teachers from other subjects, such as science and social sciences, should give students the opportunity to read and interpret data from charts, tables, and graphs.

Grades 9–10 (compare to Grade 10 in 2001 report)

- Teachers should include practice on interpretation and application of results when calculating central tendencies and range.
- Students should practice generalizing what the data mean and how they should be used.
- Students need more opportunities to interpret graphs and manipulate data to make predictions and conclusions.
- Students also should practice identifying the correct type of graph to represent data.

The DOE and task force encourage educators to use FCAT results in any way that is statistically appropriate. The comparisons of findings between the 2001 report and the 2007 report described in this section provide possibilities for evaluation at the school and district levels.

. . . the task force recommended that math educators provide students with a greater variety of contexts . . .

In reviewing this report, many educators will likely see the performance of their own students reflected in the results of the data analysis and the findings of the task force. In the data analysis and task force review of performance trends on a large sample of items, there were implications for instruction that will likely apply to many classrooms across many grades. Across grades, the task force made the following recommendations:

- Task force members saw the need for the use of manipulatives in instruction. This recommendation is tied to an emphasis on the need for students to use pictures and diagrams when working many problems.

- The task force determined that students who were not able to correctly answer some problems were likely not using appropriate reading strategies. Recommendations were to ask students to read the problem in its entirety before attempting to solve it and mark text as they read.
- The task force determined that there were a wide variety of contexts in FCAT items, which was a challenge to students who were not able to correctly answer the question. Subsequently, the task force recommended that math educators provide students with a greater variety of contexts and that educators in other content areas look for opportunities to reinforce math thinking.
- The task force determined that struggling students still appear to need help understanding the language of mathematics and being able to convert textual statements to mathematical ones. Some suggested strategies have been provided in the implications for instruction in each standard.
- Although there have been improvements in student performance in mathematics, the task force has identified many opportunities for growth. The detailed observations and implications for instruction (by strand) are intended to provide Florida educators with suggestions for enabling further improvement in student performance.

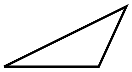
This summary information, as well as the additional details provided throughout *Lessons Learned*, are intended to provide educators with information that can further support the continuation of existing initiatives and possibly re-target programmatic change focused on addressing new issues.

Lessons Learned

FCAT MATHEMATICS REFERENCES

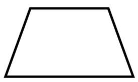
Grades 6–8 FCAT Mathematics Reference Sheet

Area

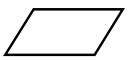


Triangle $A = \frac{1}{2}bh$

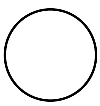
Rectangle $A = lw$



Trapezoid $A = \frac{1}{2}h(b_1 + b_2)$



Parallelogram $A = bh$



Circle $A = \pi r^2$

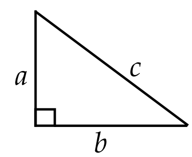
In a polygon, the sum of the measures of the interior angles is equal to $180(n - 2)$, where n represents the number of sides.

KEY	
b = base	d = diameter
h = height	r = radius
l = length	A = area
w = width	C = circumference
$S.A.$ = surface area	V = volume
Use 3.14 or $\frac{22}{7}$ for π.	

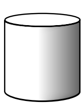
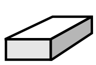
Circumference

$C = \pi d$ or $C = 2\pi r$

Pythagorean Theorem



$a^2 + b^2 = c^2$

	Volume/Capacity	Total Surface Area
 Right Circular Cylinder	$V = \pi r^2 h$	$S.A. = 2\pi rh + 2\pi r^2$
 Rectangular Prism	$V = lwh$	$S.A. = 2(lw) + 2(hw) + 2(lh)$

Conversions

1 yard = 3 feet = 36 inches
1 mile = 1760 yards = 5280 feet
1 acre = 43,560 square feet
1 hour = 60 minutes
1 minute = 60 seconds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

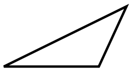
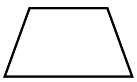
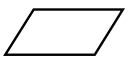
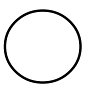
1 liter = 1000 milliliters = 1000 cubic centimeters
1 meter = 100 centimeters = 1000 millimeters
1 kilometer = 1000 meters
1 gram = 1000 milligrams
1 kilogram = 1000 grams

1 pound = 16 ounces
1 ton = 2000 pounds

Metric numbers with four digits are presented without a comma (e.g., 9960 kilometers). For metric numbers greater than four digits, a space is used instead of a comma (e.g., 12 500 liters).

Grades 9–10 FCAT Mathematics Reference Sheet

Area

	Triangle	$A = \frac{1}{2}bh$
	Rectangle	$A = lw$
	Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
	Parallelogram	$A = bh$
	Circle	$A = \pi r^2$

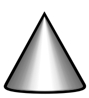
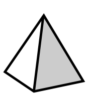
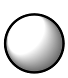
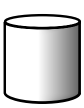
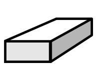
KEY

b	= base	d	= diameter
h	= height	r	= radius
l	= length	A	= area
w	= width	C	= circumference
ℓ	= slant height	V	= volume
$S.A.$	= surface area		

Use 3.14 or $\frac{22}{7}$ for π.

Circumference

$C = \pi d$ or $C = 2\pi r$

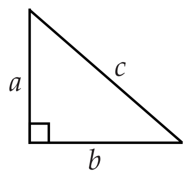
		Volume/Capacity	Total Surface Area
	Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	$S.A. = \frac{1}{2}(2\pi r)\ell + \pi r^2$ or $S.A. = \pi r\ell + \pi r^2$
	Right Square Pyramid	$V = \frac{1}{3}lwh$	$S.A. = 4(\frac{1}{2}l\ell) + l^2$ or $S.A. = 2l\ell + l^2$
	Sphere	$V = \frac{4}{3}\pi r^3$	$S.A. = 4\pi r^2$
	Right Circular Cylinder	$V = \pi r^2 h$	$S.A. = 2\pi rh + 2\pi r^2$
	Rectangular Prism	$V = lwh$	$S.A. = 2(lw) + 2(hw) + 2(lh)$

In the following formulas, n represents the number of sides.

- In a polygon, the sum of the measures of the interior angles is equal to $180(n-2)$.
- In a regular polygon, the measure of an interior angle is equal to $\frac{180(n-2)}{n}$.

Grades 9–10 FCAT Mathematics Reference Sheet

Pythagorean theorem:



$$a^2 + b^2 = c^2$$

Slope-intercept form of an equation of a line:

$$y = mx + b$$

where m = slope and b = the y-intercept.

Distance, rate, time formula:

$$d = rt$$

where d = distance, r = rate, t = time.

Distance between two points $P_1\,(x_1, y_1)$ and $P_2\,(x_2, y_2)$:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint between two points $P_1\,(x_1, y_1)$ and $P_2\,(x_2, y_2)$:

$$\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Simple interest formula:

$$\mathrm{I} = prt$$

where p = principal, r = rate, t = time.

Conversions

1 yard = 3 feet = 36 inches
1 mile = 1760 yards = 5280 feet
1 acre = 43,560 square feet
1 hour = 60 minutes
1 minute = 60 seconds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

1 liter = 1000 milliliters = 1000 cubic centimeters
1 meter = 100 centimeters = 1000 millimeters
1 kilometer = 1000 meters
1 gram = 1000 milligrams
1 kilogram = 1000 grams

1 pound = 16 ounces
1 ton = 2000 pounds

Metric numbers with four digits are presented without a comma (e.g., 9960 kilometers). For metric numbers greater than four digits, a space is used instead of a comma (e.g., 12 500 liters).

FCAT Mathematics Short-Response Rubric

THINK
SOLVE
EXPLAIN

General Short-Response Scoring Rubric

2 points A score of two indicates that the student has demonstrated a thorough understanding of the mathematics concepts and/or procedures embodied in the task. The student has completed the task correctly, in a mathematically sound manner. When required, student explanations and/or interpretations are clear and complete. The response may contain minor flaws that do not detract from the demonstration of a thorough understanding.

1 point A score of one indicates that the student has provided a response that is only partially correct. For example, the student may provide a correct solution, but may demonstrate some misunderstanding of the underlying mathematical concepts or procedures. Conversely, a student may provide a computationally incorrect solution but could have applied appropriate and mathematically sound procedures, or the student's explanation could indicate an understanding of the task, despite the error.

0 points A score of zero indicates that the student has provided no response at all, or a completely incorrect or uninterpretable response, or demonstrated insufficient understanding of the mathematics concepts and/or procedures embodied in the task. For example, a student may provide some work that is mathematically correct, but the work does not demonstrate even a rudimentary understanding of the primary focus of the task.

FCAT Mathematics Extended-Response Rubric

General Extended-Response Scoring Rubric

4 points

A score of four is a response in which the student demonstrates a thorough understanding of the mathematics concepts and/or procedures embodied in the task. The student has responded correctly to the task, used mathematically sound procedures, and provided clear and complete explanations and interpretations.

The response may contain minor flaws that do not detract from the demonstration of a thorough understanding.

3 points

A score of three is a response in which the student demonstrates an understanding of the mathematics concepts and/or procedures embodied in the task. The student's response to the task is essentially correct with the mathematical procedures used and the explanations and interpretations provided demonstrating an essential but less than thorough understanding.

The response may contain minor flaws that reflect inattentive execution of mathematical procedures or indications of some misunderstanding of the underlying mathematics concepts and/or procedures.

2 points

A score of two indicates that the student has demonstrated only a partial understanding of the mathematics concepts and/or procedures embodied in the task. Although the student may have used the correct approach to obtaining a solution or may have provided a correct solution, the student's work lacks an essential understanding of the underlying mathematical concepts.

The response contains errors related to misunderstanding important aspects of the task, misuse of mathematical procedures, or faulty interpretations of results.

1 points

A score of one indicates that the student has demonstrated a very limited understanding of the mathematics concepts and/or procedures embodied in the task. The student's response is incomplete and exhibits many flaws. Although the student's response has addressed some of the conditions of the task, the student reached an inadequate conclusion and/or provided reasoning that was faulty or incomplete.

The response exhibits many flaws or may be incomplete.

0 points

A score of zero indicates that the student has provided no response at all, or a completely incorrect or uninterpretable response, or demonstrated insufficient understanding of the mathematics concepts and/or procedures embodied in the task. For example, a student may provide some work that is mathematically correct, but the work does not demonstrate even a rudimentary understanding of the primary focus of the task.

Lessons Learned
FCAT RESOURCES

FCAT Publications and Products

The Department of Education produces many materials to help educators, students, and parents better understand the FCAT program. A list of FCAT-related publications and products is provided below. Additional information about the FCAT program is available on the FCAT home page of the DOE website at http://fcat.fldoe.org.

About the FCAT Web Brochure

This web-based brochure is found on the DOE website at http://fcat.fldoe.org/aboutfcat/english/index.html. English, Spanish, and Haitian Creole brochures provide information about FCAT Reading, Writing+, Mathematics, and Science for Grades 3–11 and link the reader to other helpful DOE web resources.

Assessment & Accountability Briefing Book

This book provides an overview of Florida's assessment, school accountability, and teacher certification programs. FCAT topics include frequently asked questions, content assessed by the FCAT, reliability, and validity. This booklet can be downloaded from the DOE website at http://fcat.fldoe.org/fcatpub1.asp.

FCAT Handbook—A Resource for Educators

This publication provides the first comprehensive look at the FCAT, including history, test content, test format, test development and construction, test administration, and test scoring and reporting. Educator involvement is emphasized, demonstrating how Florida teachers and administrators participate in reviewing test items, determining how standards should be assessed, finding ranges of scores, and providing input on aspects of the test administration process. The PDF version is available on the DOE website at http://fcat.fldoe.org/handbk/fcathandbook.asp.

FCAT Myths vs. Facts

By providing factual information about the FCAT program, this brochure addresses common concerns about the FCAT that are based on myths. It is also available in Spanish and can be downloaded from the DOE website at http://fcat.fldoe.org/fcatpub3.asp.

FCAT Performance Task Scoring—Practice for Educators (publications and software)

These materials are designed to help teachers learn to score FCAT Reading, Writing, and Mathematics performance tasks at Grades 4, 5, 8, and 10. *A Trainer's Guide* includes instructions for using the scoring publications and software in teacher education seminars and workshops. The publications mirror the scorer training experiences by presenting samples of student work for teachers to score.

FCAT Posters

Elementary, middle, and high school FCAT Reading, Writing+, Science, and Mathematics posters have an instructional focus. Two additional posters provide information about achievement levels and which FCAT tests are given at each grade. A high school poster reminds students about the graduation requirement to pass the FCAT Reading and Mathematics tests and the multiple opportunities available to retake the tests. Posters were delivered to Florida school districts in 2005; limited numbers of these posters are still available from the DOE Assessment office.

FCAT Released Tests

Reading, Grades 3, 4, 7, 8, 9, and 10
Mathematics, Grades 3, 4, 7, 8, 9, and 10

The DOE released previously used full tests of the FCAT Reading and FCAT Mathematics for Grades 4, 8, and 10 in 2005 and for Grades 3, 7, 9, and 10 in 2006. This web-based release included not only the tests, but also several other important documents including interactive test books, answer keys, *How to Use the FCAT Released Tests, How to Score the FCAT Released Tests,* and *Frequently Asked Questions About the FCAT Released Tests*. These supplemental materials provide many details about the FCAT that are informative for all audiences, especially the range of correct answers and points needed for each achievement level. All materials are available on the DOE website at http://fcat.fldoe.org/fcatrelease.asp. In 2007 the DOE plans to release FCAT Reading and FCAT Mathematics tests for Grades 5 and 6.

FCAT Results Folder: A Guide for Parents and Guardians

This folder is designed for parents and guardians of students in Grades 3–11. It provides information about FCAT student results and allows parents to store student reports for future reference. Spanish and Haitian Creole versions are available. Delivery coincides with the spring delivery of student reports.

FCAT Test Item Specifications

Reading, Grade Levels 3–5, 6–8, and 9–10
Mathematics, Grade Levels 3–5, 6–8, and 9–10
Science, Grades 5, 8, and 10/11
Writing+ draft versions, Grades 4, 8, and 10

Defining both the content and the format of the FCAT test questions, the *Specifications* primarily serve as guidelines for item writers and reviewers, but also contain information for educators and the general public. The *Specifications* are designed to be broad enough to ensure test items are developed in several formats to measure the concepts presented in each benchmark. These materials can be downloaded from the DOE website at http://fcat.fldoe.org/fcatis01.asp.

Florida Reads! Report on the 2007 FCAT Reading Released Items (Grades 4, 8 & 10)
Florida Solves! Report on the 2007 FCAT Mathematics Released Items (Grades 5, 8 & 10)
Florida Inquires! Report on the 2007 FCAT Science Released Items (Grades 5, 8 & 11)

These reports provide information about the scoring of the FCAT Reading, Mathematics, and Science performance tasks displayed on the 2007 student reports. *Florida Reads!* combines Grades 4, 8, and 10 in one document; *Florida Solves!* covers Grades 5, 8, and 10; and *Florida Inquires!* includes Grades 5, 8, and 11. The reports are distributed each May and are also posted to the DOE website at http://fcat.fldoe.org/fcatflwrites.asp.

Florida Writes! Report on the 2007 FCAT Writing+ Assessment, Grade 4
Florida Writes! Report on the 2007 FCAT Writing+ Assessment, Grade 8
Florida Writes! Report on the 2007 FCAT Writing+ Assessment, Grade 10

Each grade-level publication describes the content and application of the prompt portion of the FCAT Writing+ tests and offers suggestions for activities that may be helpful in preparing students for the draft writing assessment. The reports are distributed each May and are also posted to the DOE website at http://fcat.fldoe.org/fcatflwrites.asp.

Frequently Asked Questions About FCAT

This brochure provides answers to frequently asked questions about the FCAT program and is available on the DOE website at http://fcat.fldoe.org/fcatpub3.asp.

Keys to FCAT, Grades 3–5, 6–8, and 9–11

These booklets are distributed each January and contain information for parents and students preparing for FCAT Reading, Writing+, Mathematics, and Science. *Keys to FCAT* are translated into Spanish and Haitian Creole and are available, along with the English version, on the DOE website at http://fcat.fldoe.org/fcatkeys.asp.

Lessons Learned—FCAT, Sunshine State Standards and Instructional Implications

The original *Lessons Learned* provides analyses of FCAT Reading, Writing, and Mathematics results based on state-level data through 2000. The PDF version is available on the DOE website at http://fcat.fldoe.org/fclesn02.asp. This publication, *FCAT Mathematics Lessons Learned: 2001–2005 Data Analysis and Instructional Implications*, is the next in the *Lessons Learned* series. The companion volume, *FCAT Reading Lessons Learned: 2001–2005 Data Analysis and Instructions Implications*, is being released at the same time and is also available on the DOE website.

Sample Test Materials for the FCAT
Reading and Mathematics, Grades 3–10
Science, Grades 5, 8, and 11
Writing+, Grades 4, 8, and 10

These materials are produced and distributed each fall for teachers to use with students. The student's test booklet contains practice questions and hints for answering them. The teacher's answer key provides the correct answer, an explanation for the correct answer, and also indicates the assessed SSS benchmark. These booklets are available in PDF format on the DOE website at http://fcat.fldoe.org/fcatsmpl.asp.

The New FCAT NRT: Stanford Achievement Test, Tenth Edition (SAT10)

This brochure outlines differences between the previous FCAT NRT (SAT9) and the current FCAT NRT (SAT10). It is available in PDF format on the DOE website at http://fcat.fldoe.org/fcatpub2.asp.

Understanding FCAT Reports

This booklet provides information about the FCAT student, school, and district reports for the recent test administration. Sample reports, explanations about the reports, and a glossary of technical terms are included. Distribution to districts is scheduled to coincide with the delivery of student reports each May. The booklet can be downloaded from the DOE website at http://fcat.fldoe.org/fcatpub2.asp.

What every teacher should know about FCAT

This document provides suggestions for all subject-area teachers to use in helping their students be successful on the FCAT. It can be downloaded from the DOE website at http://fcat.fldoe.org/fcatpub2.asp.

Online FCAT Data Resources

FCAT Scores and Reports for schools, districts, and the state for all test administrations are accessible from the DOE website: http://fcat.fldoe.org/fcatscor.asp.

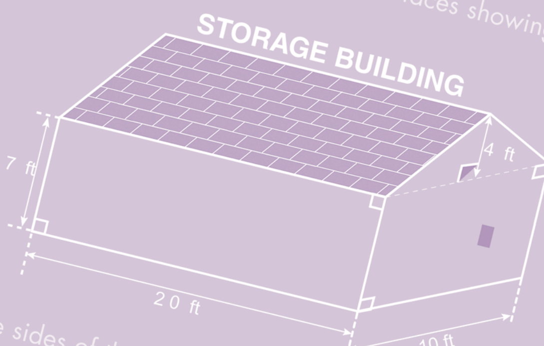
FCAT Technical Reports are available at the DOE website: http://fcat.fldoe.org/fcatpub2.asp.

The FCAT Demographic Results website, located at http://www.fcatresults.com/demog/, provides searches on school-, district-, and state-level data. The site currently hosts data for FCAT Reading and Mathematics 2000 through 2007 and FCAT Writing 2001 through 2007.

2004–2005 School Grades and Accountability Reports are available at http://schoolgrades.fldoe.org/ on the DOE website.

THINK
SOLVE
EXPLAIN

Paolo and Fred need to paint the 4 outside faces of a storage building. Before they can purchase the paint, they must calculate the surface area of the faces to be painted. A diagram of the building with 2 outside faces showing is given below.



Assuming that opposite sides of the building are congruent to each other, what is the total outside surface area, in square feet, of the 4 faces to be painted? Show all work necessary to justify your answer.

An explanation similar to the following:

Surface area of the front and back faces:

$2(20 \times 7) = 280$

Surface area of the two end faces:

$2(10 \times 7) = 140$

Surface area of the two triangular sections:

$2\left(\frac{1}{2}\right)(10 \times 4) = 40$

$280 + 140 + 40 = 460$

Total Surface Area of Outside Faces 460 square feet

Florida Department of Education
www.fldoe.org

ISBN 999-8570-65-4

9 789998 570658

Assessment and School Performance
Florida Department of Education
Tallahassee, Florida