

**Mathematics Florida Standards (MAFS)
Grades 9–12 Calculus**

Standard 1: Limits and Continuity	
Develop an understanding of the concept of limit by estimating limits graphically and numerically and evaluating limits analytically. Extend the idea of a limit to one-sided limits and limits at infinity. Use limits to define and understand the concept of continuity, decide whether a function is continuous at a point, and find types of discontinuities. Understand and apply continuity theorems.	
BENCHMARK CODE	BENCHMARK
MAFS.912.C.1.1	Understand the concept of limit and estimate limits from graphs and tables of values. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.1.10	Decide if a function is continuous at a point. <i>Cognitive Complexity:</i> Level 3: Strategic Thinking & Complex Reasoning
MAFS.912.C.1.11	Find the types of discontinuities of a function. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.1.12	Understand and use the Intermediate Value Theorem on a function over a closed interval. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.1.13	Understand and apply the Extreme Value Theorem: If $f(x)$ is continuous over a closed interval, then f has a maximum and a minimum on the interval. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.1.2	Find limits by substitution. <i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.1.3	Find limits of sums, differences, products, and quotients. <i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.1.4	Find limits of rational functions that are undefined at a point. <i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.1.5	Find one-sided limits. <i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.1.6	Find limits at infinity. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.1.7	Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts

MAFS.912.C.1.8	Find special limits such as $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.1.9	Understand continuity in terms of limits. <i>Cognitive Complexity:</i> Level 3: Strategic Thinking & Complex Reasoning

Standard 2: Differential Calculus

Develop an understanding of the derivative as an instantaneous rate of change, using geometrical, numerical, and analytical methods. Use this definition to find derivatives of algebraic and transcendental functions and combinations of these functions (using, for example, sums, composites, and inverses). Find second and higher order derivatives. Understand and use the relationship between differentiability and continuity. Understand and apply the Mean Value Theorem. Find derivatives of algebraic, trigonometric, logarithmic, and exponential functions. Find derivatives of sums, products, and quotients, and composite and inverse functions. Find derivatives of higher order, and use logarithmic differentiation and the Mean Value Theorem.

BENCHMARK CODE	BENCHMARK
MAFS.912.C.2.1	Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as an instantaneous rate of change or as the slope of the tangent line. <i>Cognitive Complexity:</i> Level 3: Strategic Thinking & Complex Reasoning
MAFS.912.C.2.10	Understand and use the relationship between differentiability and continuity. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.2.11	Understand and apply the Mean Value Theorem. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.2.2	State, understand, and apply the definition of derivative. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.2.3	Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions. <i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.2.4	Find the derivatives of sums, products, and quotients. <i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.2.5	Find the derivatives of composite functions using the Chain Rule. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.2.6	Find the derivatives of implicitly-defined functions. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.2.7	Find derivatives of inverse functions.

	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.2.8	Find second derivatives and derivatives of higher order. <i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.2.9	Find derivatives using logarithmic differentiation. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts

Standard 3: Applications of Derivatives

Apply knowledge about derivatives to find slopes of curves and the related tangent lines. Analyze and graph functions, finding where they are increasing or decreasing, their maximum and minimum points, their points of inflection, and their concavity. Solve optimization problems, find average and instantaneous rates of change (including velocities and accelerations), and model rates of change. Find slopes and equations of tangent lines, maximum and minimum points, and points of inflection. Solve optimization problems, and find rates of change.

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MAFS.912.C.3.1	Find the slope of a curve at a point, including points at which there are vertical tangent lines and no tangent lines. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.3.10	Find the velocity and acceleration of a particle moving in a straight line. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.3.11	Model rates of change, including related rates problems. <i>Cognitive Complexity:</i> Level 3: Strategic Thinking & Complex Reasoning
MAFS.912.C.3.12	Solve problems using the Newton-Raphson method. <i>Cognitive Complexity:</i> Level 3: Strategic Thinking & Complex Reasoning
MAFS.912.C.3.2	Find an equation for the tangent line to a curve at a point and a local linear approximation. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.3.3	Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of f and the sign of f' . <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.3.4	Find local and absolute maximum and minimum points. <i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.3.5	Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f'' . Understand points of inflection as places where concavity changes.

	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.3.6	Use first and second derivatives to help sketch graphs. Compare the corresponding characteristics of the graphs of f , f' , and f'' .
	<i>Cognitive Complexity:</i> Level 3: Strategic Thinking & Complex Reasoning
MAFS.912.C.3.7	Use implicit differentiation to find the derivative of an inverse function.
	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.3.8	Solve optimization problems.
	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.3.9	Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed, and acceleration.
	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts

Standard 4: Integral Calculus

Understand that integration is used to find areas, and evaluate integrals using rectangular approximations. From this, develop the idea that integration is the inverse operation to differentiation — the Fundamental Theorem of Calculus. Use this result to find definite and indefinite integrals, including using the method of integration by substitution. Apply approximate methods, such as the Trapezoidal Rule, to find definite integrals. Define integrals using Riemann sums, use the Fundamental Theorem of Calculus to find integrals using antiderivatives, and use basic properties of integrals. Integrate by substitution, and find approximate integrals.

BENCHMARK CODE	BENCHMARK
MAFS.912.C.4.1	Use rectangle approximations to find approximate values of integrals.
	<i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.4.2	Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points.
	<i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.4.3	Interpret a definite integral as a limit of Riemann sums.
	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.4.4	Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval. That is, $\int_a^b f'(x)dx = f(b) - f(a)$ (Fundamental Theorem of Calculus).
	<i>Cognitive Complexity:</i> Level 3: Strategic Thinking & Complex Reasoning
MAFS.912.C.4.5	Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined.

	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.4.6	<p>Use these properties of definite integrals:</p> <ul style="list-style-type: none"> $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ $\int_a^b k \cdot f(x)dx = k \int_a^b f(x)dx$ $\int_a^a f(x)dx = 0$ $\int_a^b f(x)dx = - \int_b^a f(x)dx$ $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ If $f(x) \leq g(x)$ on $[a, b]$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$ <p><i>Cognitive Complexity:</i> Level 1: Recall</p>
MAFS.912.C.4.7	<p>Use integration by substitution (or change of variable) to find values of integrals.</p> <p><i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts</p>
MAFS.912.C.4.8	<p>Use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.</p> <p><i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts</p>

Standard 5: Applications of Integration	
Apply knowledge about integrals to finding velocities from accelerations, solving separable differential equations, and finding areas and volumes. Apply integration to model, and solve problems in physics, biology, economics, etc. Find velocity functions and position functions from their derivatives, solve separable differential equations, and use definite integrals to find areas and volumes.	
BENCHMARK CODE	BENCHMARK
MAFS.912.C.5.1	<p>Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and solving applications related to motion along a line.</p> <p><i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts</p>
MAFS.912.C.5.2	<p>Solve separable differential equations, and use them in modeling.</p> <p><i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts</p>
MAFS.912.C.5.3	<p>Solve differential equations of the form $\frac{dy}{dt} = ky$ as applied to growth and decay problems.</p> <p><i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts</p>
MAFS.912.C.5.4	<p>Use slope fields to display a graphic representation of the solution to a differential equation, and locate particular solutions to the equation.</p>

	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.5.5	Use definite integrals to find the area between a curve and the x-axis or between two curves.
	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts
MAFS.912.C.5.6	Use definite integrals to find the average value of a function over a closed interval.
	<i>Cognitive Complexity:</i> Level 1: Recall
MAFS.912.C.5.7	Use definite integrals to find the volume of a solid with known cross-sectional area, including solids of revolution.
	<i>Cognitive Complexity:</i> Level 3: Strategic Thinking & Complex Reasoning
MAFS.912.C.5.8	Apply integration to model, and solve problems in physical, biological, and social sciences.
	<i>Cognitive Complexity:</i> Level 2: Basic Application of Skills & Concepts

